POD/MAC-Based Modal Basis Selection for a Reduced Order Nonlinear Response Analysis

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Probabilistic Analysis and Design Tools for High-Cycle Fatigue Resistant Hypersonic Vehicle Structures
NNX07AC26A

Motivation: High Cycle Fatigue Analysis

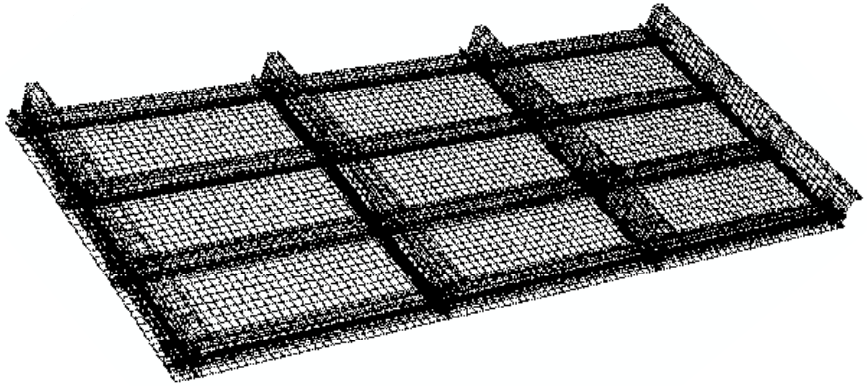
- **Fatigue life estimates require long-time stress/strain histories**
- **Analysis methods must be capable of handling complex structures under combined high-intensity thermal and acoustic environments**
  - Large deflection nonlinear response
  - Composite material properties
  - Spatially and temporally complicated loadings
- **Computationally efficient methods are required**
- **Reduced order modeling is one approach**

Representative problem of interest

58 x 25 in. multi-bay panel with stiffeners

Mixed element model

Approximately 16,000 nodes (96,000 DoFs)



Objective

- Prior work has shown that *careful selection* of normal modes as the basis functions for reduced order models results in accurate approximations

- Development of a *robust modal* basis *selection criterion* for reduced order nonlinear simulation of random response is in progress

  - Proper Orthogonal Decomposition (POD)

    → accuracy

  - Modal Assurance Criteria (MAC)

    → applicability to a wide range of loading conditions

Outline

- Nonlinear Reduced Order Formulation
- Modal Basis Selection Procedure
- Numerical Results
- Concluding Remarks

Nonlinear Reduced Order Formulation

- **Semi-discrete equations of motion in physical DoF (nDoF)**

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t)+\boldsymbol{C}\dot{\boldsymbol{X}}(t)+\underbrace{\boldsymbol{K}(\boldsymbol{X}(t))\boldsymbol{X}(t)}_{\boldsymbol{F}_{NL}(\boldsymbol{X}(t))}=\boldsymbol{F}(t)$$

nonlinear restoring force

- **Modal transformation**

$$\boldsymbol{X}(t)=\Phi\boldsymbol{q}(t)$$

Φ = matrix of basis functions
$q(t)$ = modal coordinates

- **Equation of motion in reduced-order DoF (L << nDoF)**

$$\tilde{\boldsymbol{M}}\ddot{\boldsymbol{q}}(t)+\tilde{\boldsymbol{C}}\dot{\boldsymbol{q}}(t)+\underline{\tilde{\boldsymbol{F}}_{NL}(q_1(t),q_2(t),\ldots,q_L(t))}=\tilde{\boldsymbol{F}}(t)$$

where modal quantities are

$$\tilde{\boldsymbol{M}}=\Phi^T\boldsymbol{M}\Phi=[\boldsymbol{I}] \qquad \tilde{\boldsymbol{C}}=\Phi^T\boldsymbol{C}\Phi=[2\zeta\omega] \qquad \tilde{\boldsymbol{F}}_{NL}=\Phi^T\boldsymbol{F}_{NL} \qquad \tilde{\boldsymbol{F}}=\Phi^T\boldsymbol{F}$$

Nonlinear Reduced Order Formulation

$$\tilde{M}\ddot{q} + \tilde{C}\dot{q} + \tilde{F}_{NL} = \tilde{F}$$

For each component of nonlinear restoring modal force vector

$$\tilde{F}^r_{NL}(q_1, q_2, \ldots, q_L) = \underbrace{\sum_{j=1}^{L} d^r_j q_j}_{\text{linear}} + \underbrace{\sum_{j=1}^{L}\sum_{k=1}^{L} a^r_{jk} q_j q_k}_{\text{quadratic}} + \underbrace{\sum_{j=1}^{L}\sum_{k=1}^{L}\sum_{\ell=1}^{L} b^r_{jk\ell} q_j q_k q_\ell}_{\text{cubic}}$$

- **Construct a sufficient number of *static* displacement fields**

$$q \rightarrow X = \Phi\, q$$

- **Obtain restoring forces via *static nonlinear* FE solution**

$$K(X)\, X = F \rightarrow F_{NL} \rightarrow \tilde{F}_{NL} = \Phi^T F_{NL}$$

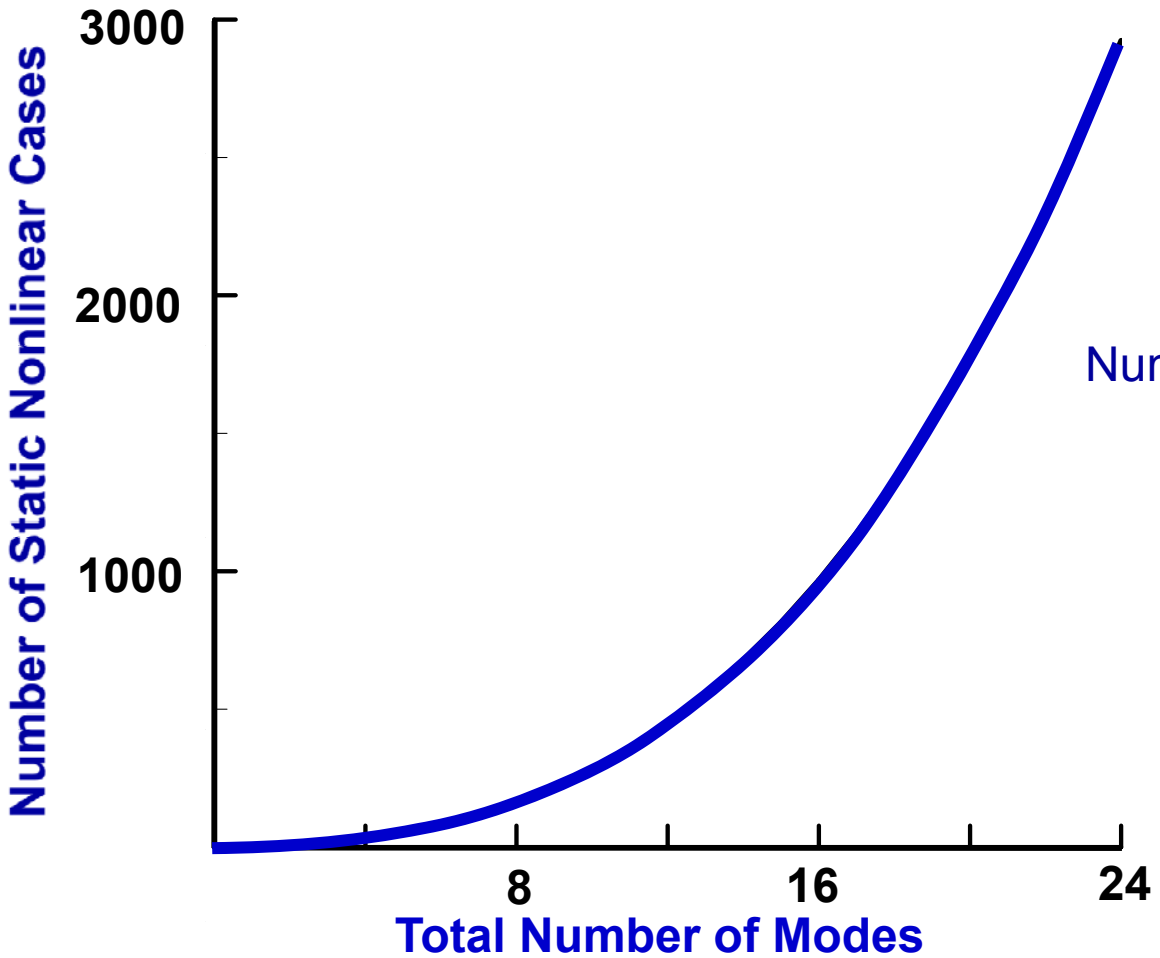
- **Solve *algebraic system* of equations for coefficients**

$$d^r_j,\ j = 1, \ldots, L \qquad a^r_{jk},\ j,k = 1, \ldots, L \qquad b^r_{jk\ell},\ j,k,\ell = 1, \ldots, L$$

"Work" to Obtain Reduced Nonlinear System

$$\tilde{F}^r_{NL}(q_1, q_2, \ldots, q_L) = \sum_{j=1}^{L} d^r_j q_j + \sum_{j=1}^{L}\sum_{k=1}^{L} a^r_{jk} q_j q_k + \sum_{j=1}^{L}\sum_{k=1}^{L}\sum_{\ell=1}^{L} b^r_{jk\ell} q_j q_k q_\ell$$

$$r = 1, \ldots, L$$



Number of Static Nonlinear Cases

$$3\binom{L}{1} + 3\binom{L}{2} + \binom{L}{3}$$

where

$$\binom{L}{n} = \frac{L!}{n!(L-n)!}$$

Modal Basis Selection Procedure

$$\tilde{\boldsymbol{M}}\ddot{\boldsymbol{q}}(t)+\tilde{\boldsymbol{C}}\dot{\boldsymbol{q}}(t)+\tilde{\boldsymbol{F}}_{NL}(q_1(t),q_2(t),\ldots,q_L(t))=\tilde{\boldsymbol{F}}(t)$$

$$\boldsymbol{X}(t)=\Phi\,\boldsymbol{q}(t)$$ **How is Φ selected?**

- **Perform system identification to *characterize* nonlinear dynamic response for a *particular* load case**
  - Proper Orthogonal Decomposition (POD) $\longrightarrow$ Proper Orthogonal Modes (POM)
- **Determine most *significant* proper orthogonal modes (POM)**
  - Cumulative POM Participation
- **Select normal modes that *best represent* proper orthogonal modes – normal modes are independent of loads**
  - Modal Assurance Criteria (MAC)

System Identification to Characterize Nonlinear Response

Proper Orthogonal Decomposition

- **Obtain sample of full-order <u>nonlinear</u> response in physical DoF for some $t_n << t_f$**

$$[X]_{n\times nDoF} = [\mathbf{X}(t_1)\ \mathbf{X}(t_2)\ \mathbf{X}(t_3)\ldots\ \mathbf{X}(t_n)]$$

- **Form correlation matrix $[R]$**

$$[R]_{nDoF\times nDoF} = \frac{1}{n}[X]^T[X]$$

- **Perform eigen-analysis of correlation matrix $[R]$**

$$\underbrace{[\lambda]}_{\text{POV's}} \longrightarrow [R - \lambda_i I]\{p_i\} = \{0\} \longrightarrow \underbrace{[\{p_1\}\quad \{p_2\}\cdots\{p_{nDoF}\}]}_{\text{Proper Orthogonal Modes (POM's)}}$$

Selection of Most Significant Proper Orthogonal Modes

- **Determine proper orthogonal mode participation – normalized POV's**

$$\chi_i = \frac{\lambda_i}{\sum_{j=1}^{nDoF} \lambda_j} \qquad i = 1, \ldots, nDoF$$

- **Specify desired level of cumulative POM participation**

$$\upsilon = \sum_{i=1}^{M} \chi_i \qquad 0 < \upsilon \leq 1 \qquad M = \#\, of\ selected\ POVs$$

where

The number of selected POM's, M, is determined by user specification of υ

Selection of Modal Basis Functions for Reduced Order Analysis

Find normal modes that *best represent* proper orthogonal modes

- **Perform normal modes analysis of *undamped linear* system**

$$\left[K - \omega_i^{\,2} M\right]\{\phi_i\} = \{0\} \qquad i = 1,\ldots,nDoF$$

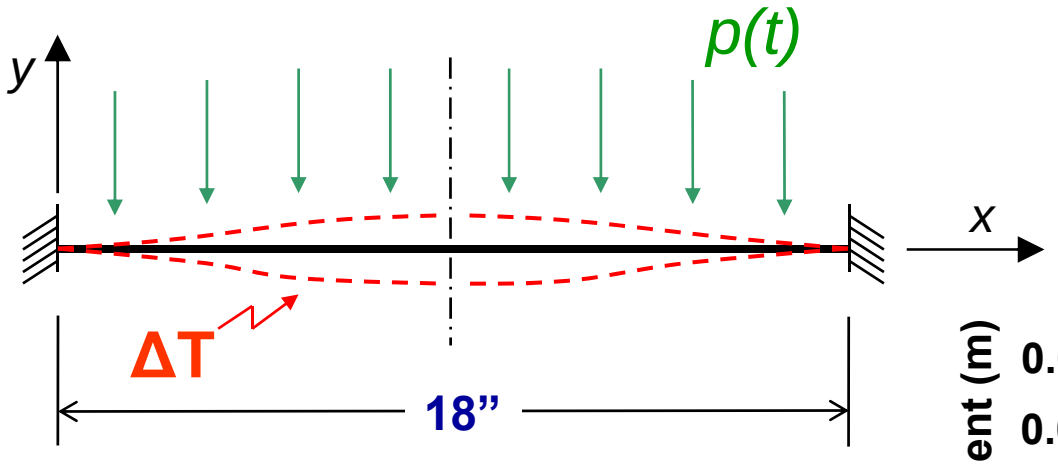
$$\lceil\omega^2\rfloor \qquad [\Phi]_{nDOF \times nDoF} = \left[\{\phi_1\}\ \{\phi_2\}\ \cdots\ \{\phi_{nDoF}\}\right]$$

- **Form modal assurance criterion (MAC) matrix for basis selection**

$$MAC(\{p_k\},\{\phi_\ell\}) = \frac{\left|\{p_k\}^T\{\phi_\ell\}\right|^2}{\left(\{p_k\}^T\{p_k\}\right)\left(\{\phi_\ell\}^T\{\phi_\ell\}\right)} \qquad \begin{array}{l} k = 1,\ldots,M \\ \ell = 1,\ldots,nDoF \end{array}$$

- **Choose normal modes with MAC > 0.5** $\longrightarrow$ Φ

Numerical Example
Clamped Aluminum Beam



1” *x* 0.9” cross-section

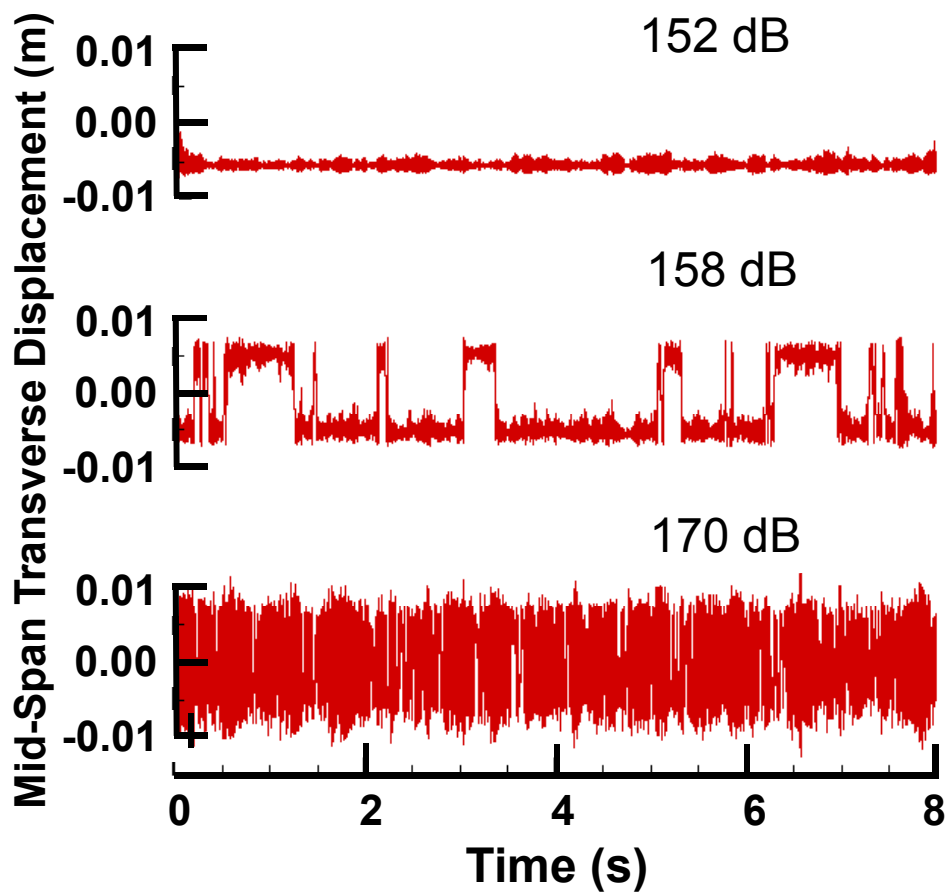
144 B21 elements

ΔT_{cr} = 6.6°F

Transverse Displacement

Post-buckled Random Response

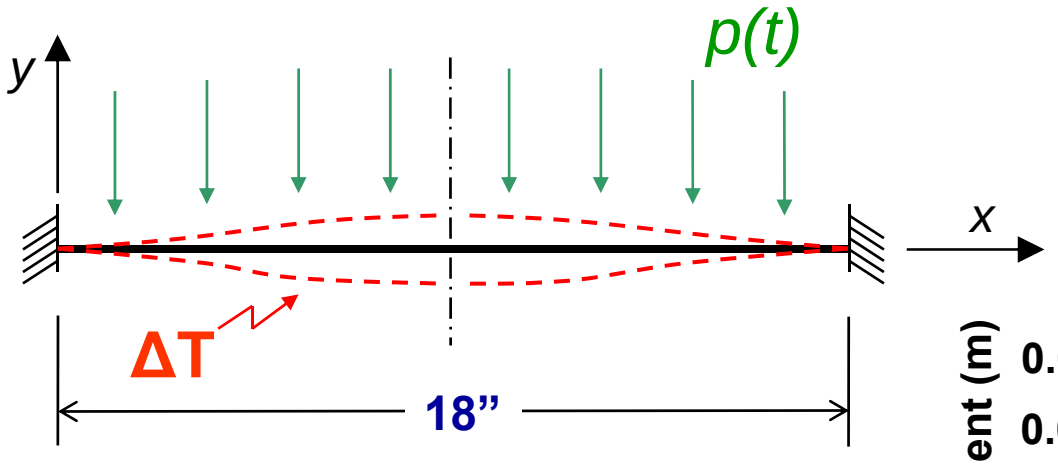
ΔT = 35°F



Excitation

- **Uniformly distributed, band-limited white noise pressure (0 – 1500 Hz)**
- **Uniform temperature distribution**

Numerical Example
Clamped Aluminum Beam



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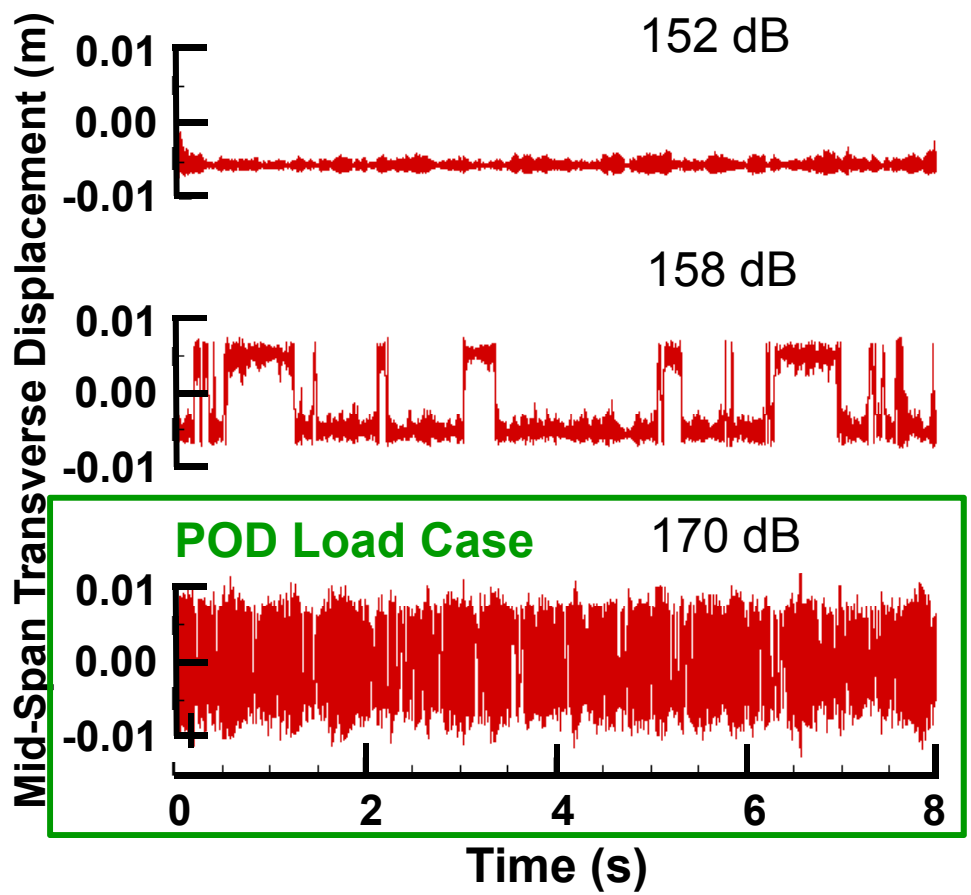

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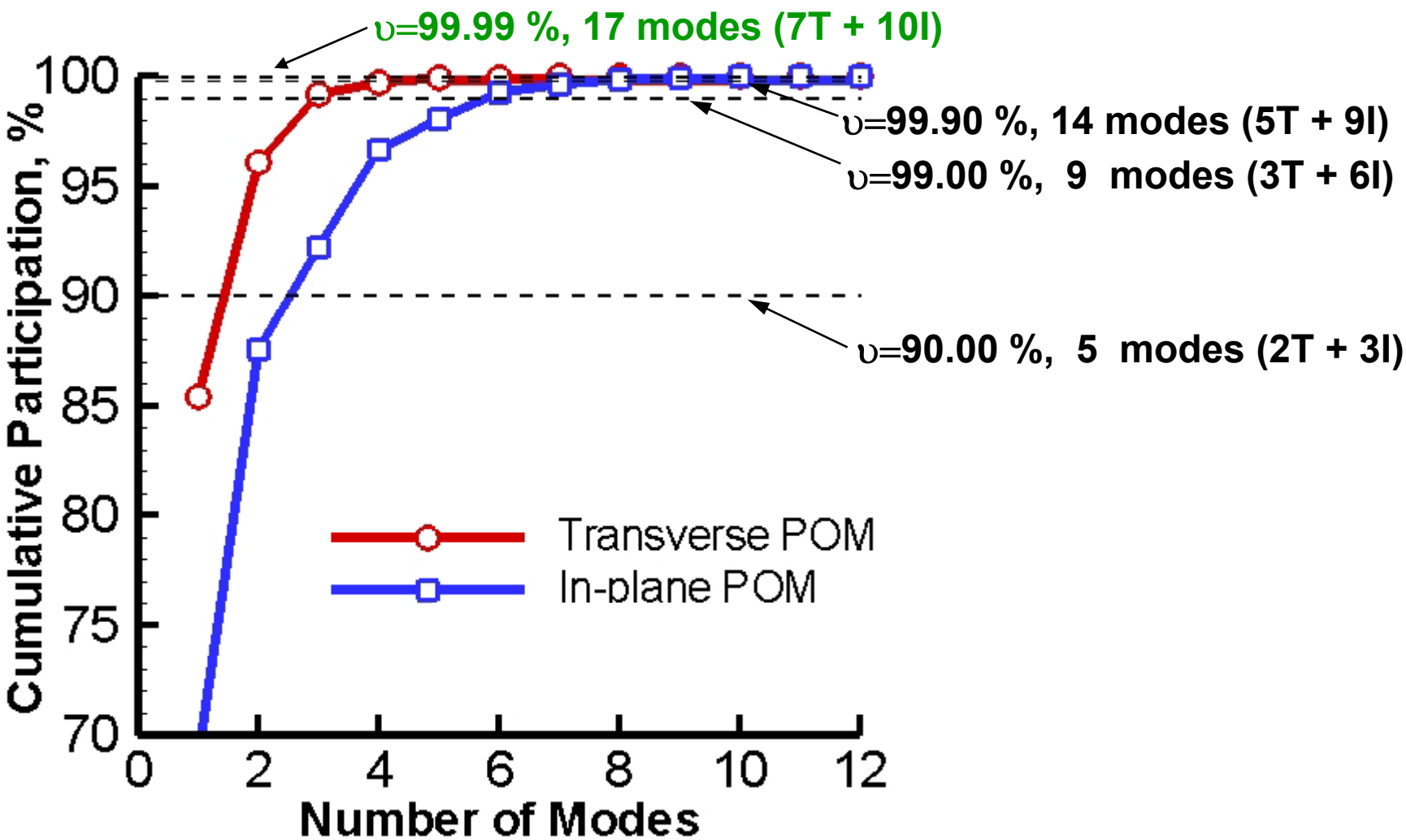
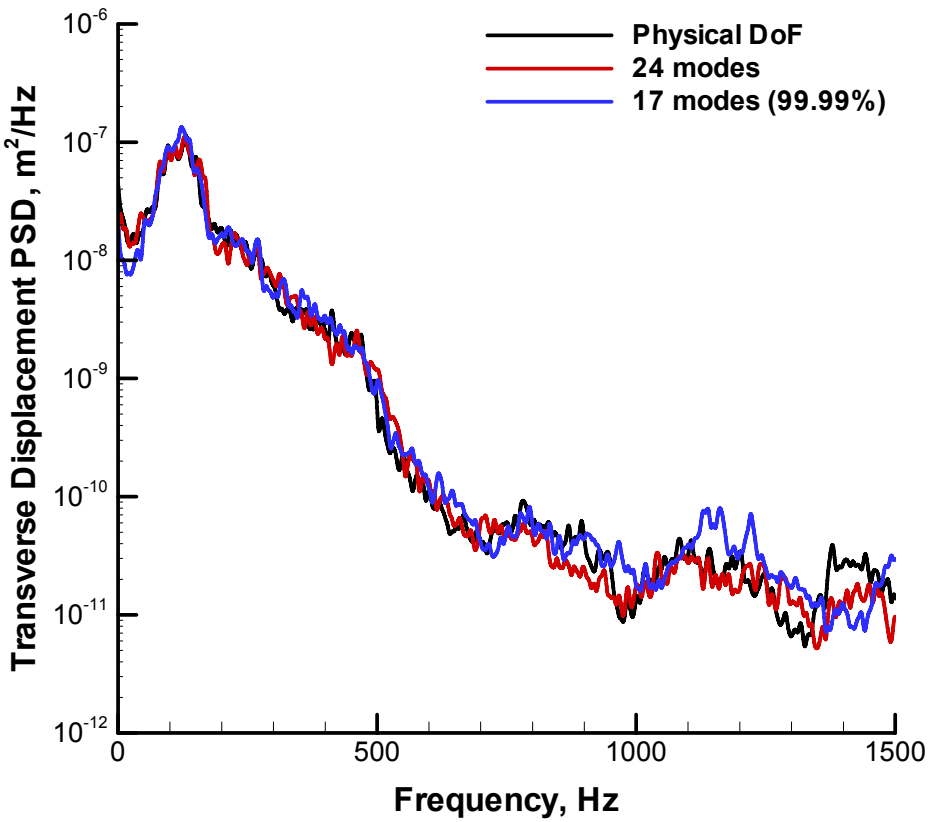
POM Selection using Cumulative Participation

POD Results for 35°F and 170 dB

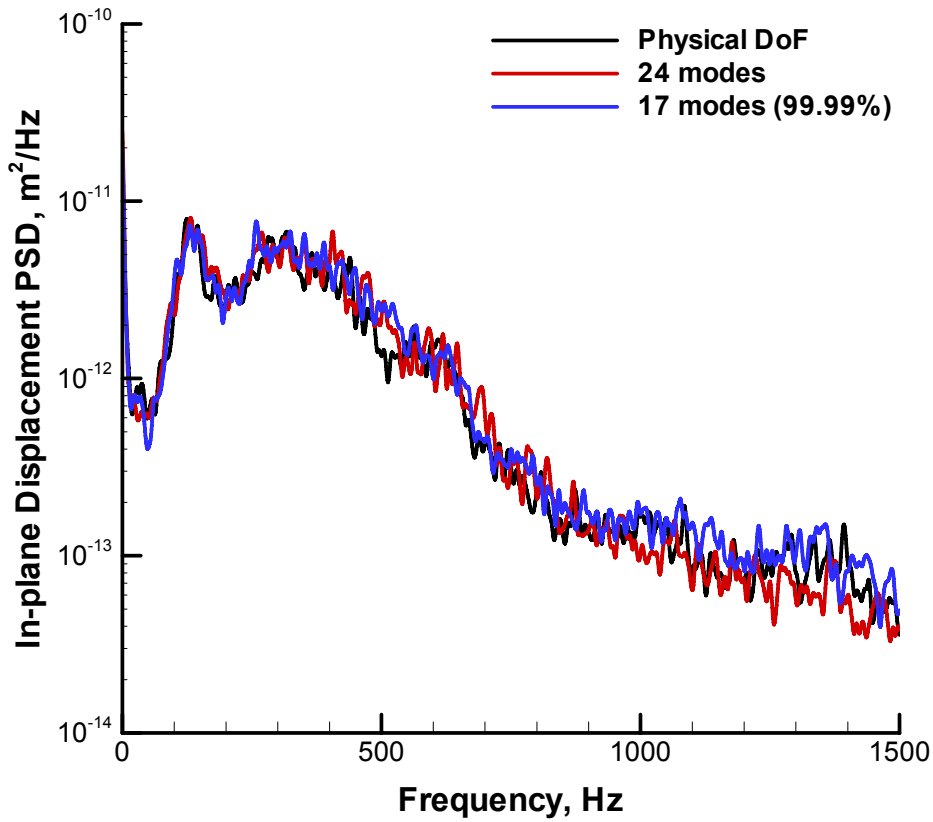


Response at Quarter-span Location for POD Load Case (35°F and 170 dB)

Transverse Response



In-Plane Response



Reduced order analysis accurately captures nonlinear response

Computational Efficiency of POD/MAC Modal Basis Selection Procedure

- **Reduced order simulation cost**
  - **System reduction**

    Metric: *Number of static nonlinear cases*

    24 modes → 2,924

    17 modes → 1,139

    =► 61% reduction
  - **Reduced system integration**

    Metric: *Runge-Kutta 4th order scheme FLOPS/time step*

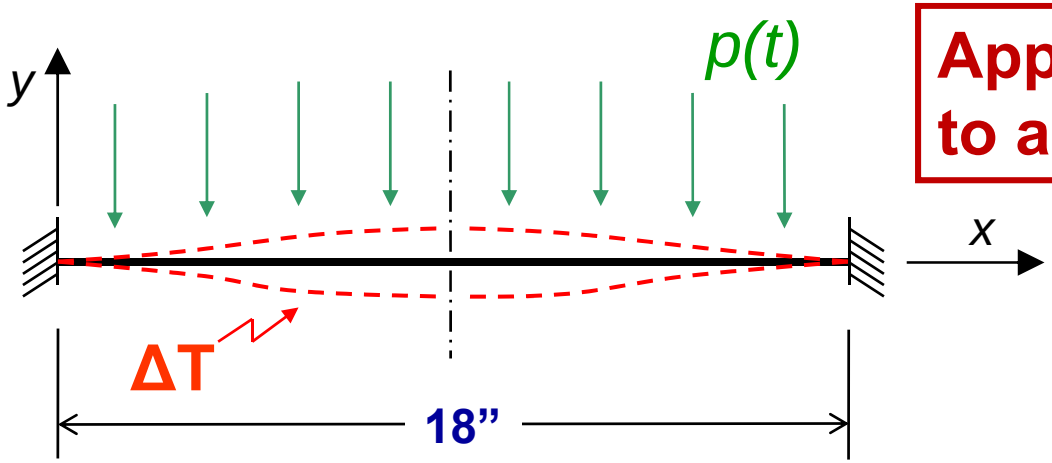
    24 modes → ~126k

    17 modes → ~45k

    =► 64 % reduction

Milestone metric of 50% computational cost reduction achieved.

Robustness Study



Apply reduced order analysis to alternate acoustic load level

1” *x* 0.9” cross-section

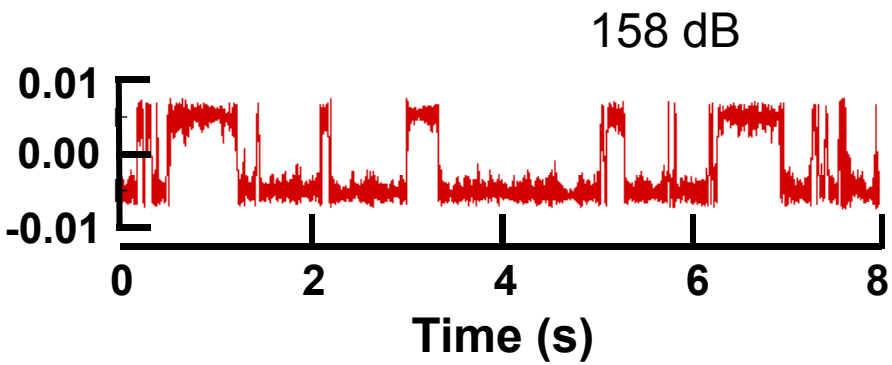
144 B21 elements (> 400 DoFs)

ΔT_{cr} = 6.6°F

Transverse Displacement

Post-buckled Random Response

ΔT = 35°F



Excitation

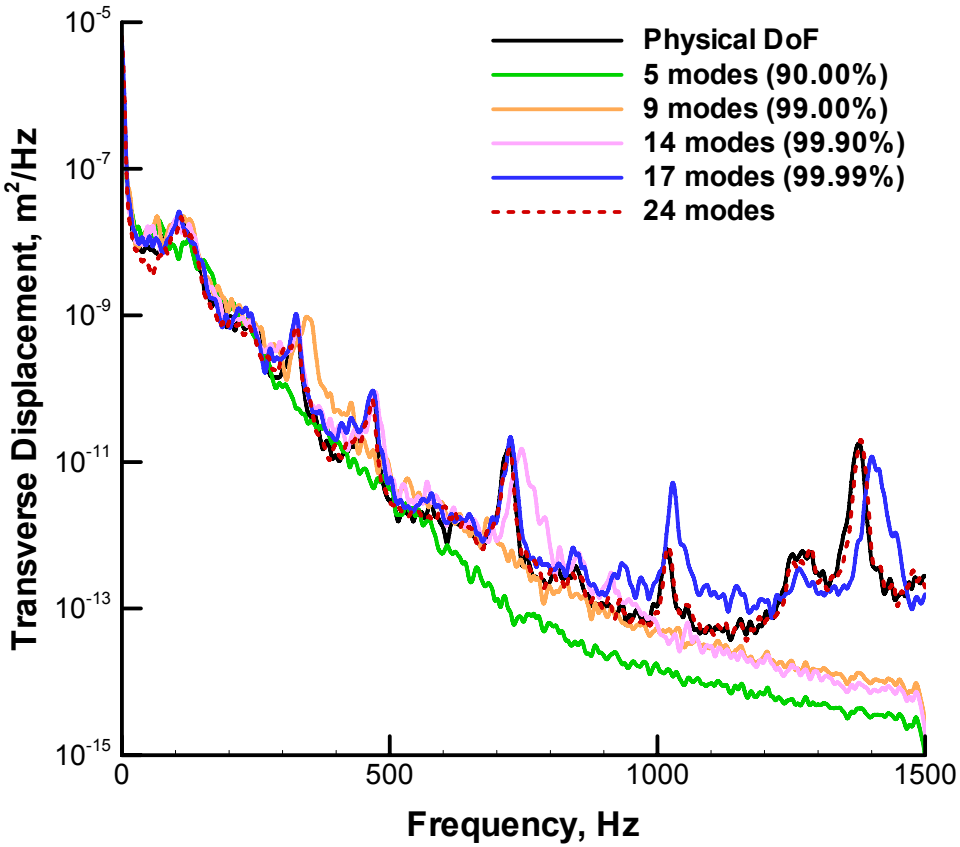
- **Uniformly distributed, band-limited white noise pressure (0 – 1500 Hz)**
- **Uniform temperature distribution**

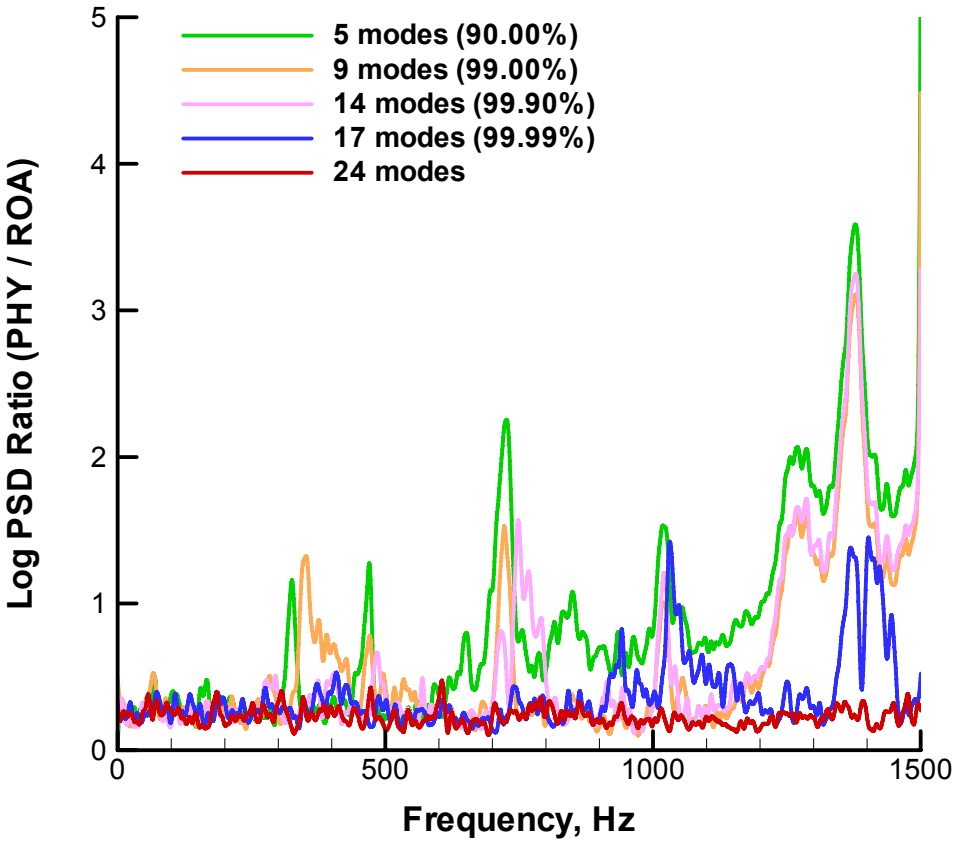
Robustness Study:

Nonlinear Response at a Reduced Thermal-Acoustic Loading Level (35°F and 158 dB)

- **Reduced order analysis derived from 35°F and 170 dB loading level**

Quarter-span location, transverse response



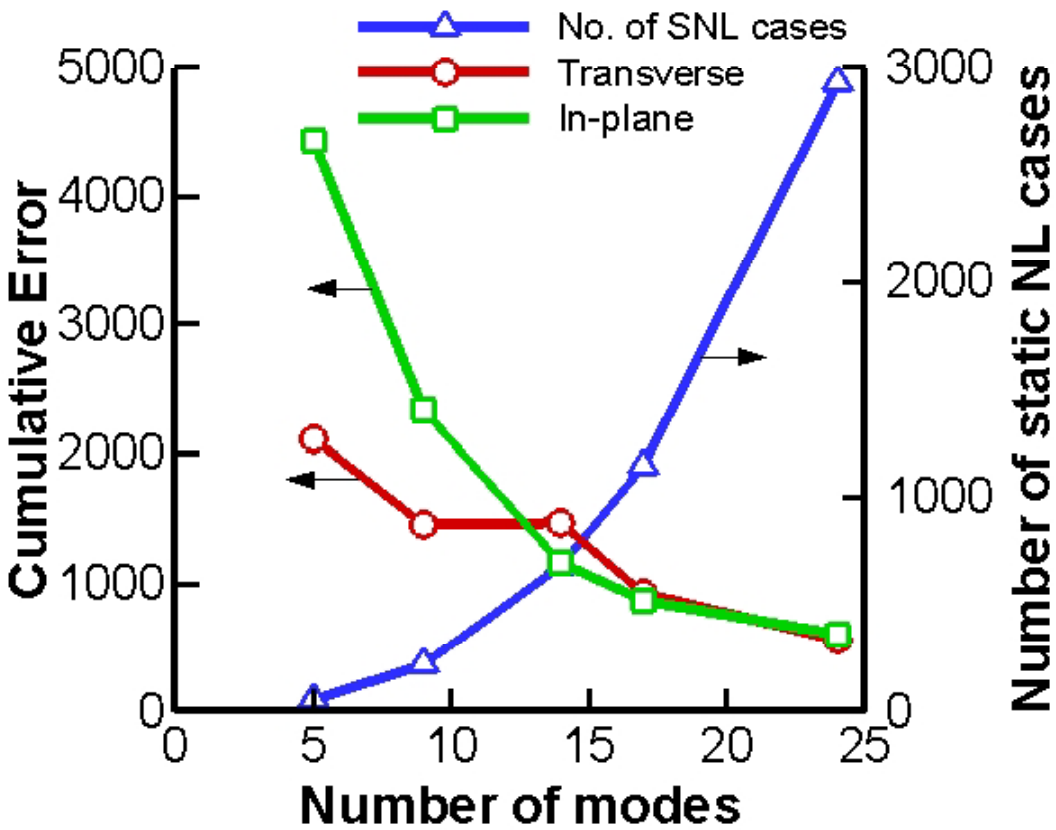


Robustness Study:

Nonlinear Response at a Reduced Acoustic Loading Level (35°F and 158 dB)

Reduced order analysis derived from 35°F and 170 dB loading level

Convergence of the error at the quarter-span location

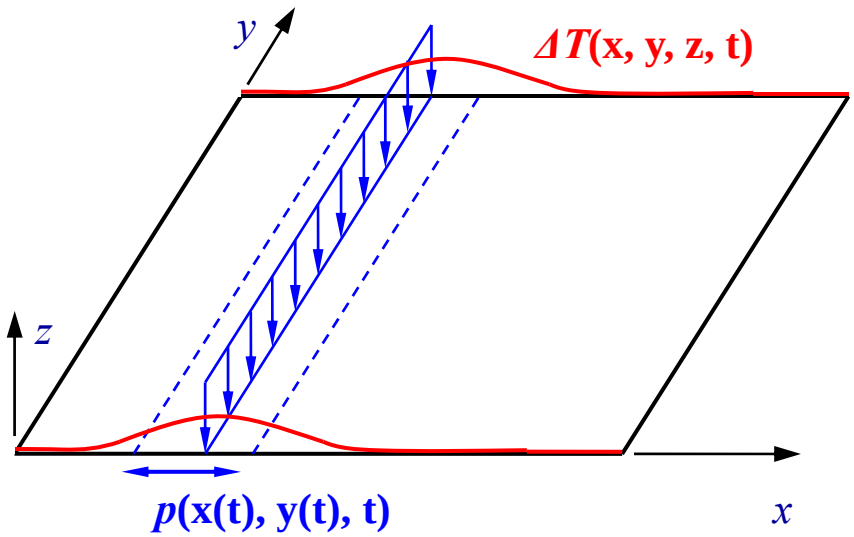


Summary

- **Modal basis selection approach using POD/MAC developed and successfully applied**
  - Error quantified as function of frequency and as a single cumulative metric
  - Relationship between cumulative error and computational effort determined

Future Work

- **Improved analysis:** Efficient handling of loads typical to a full mission profile of a hypersonic vehicle



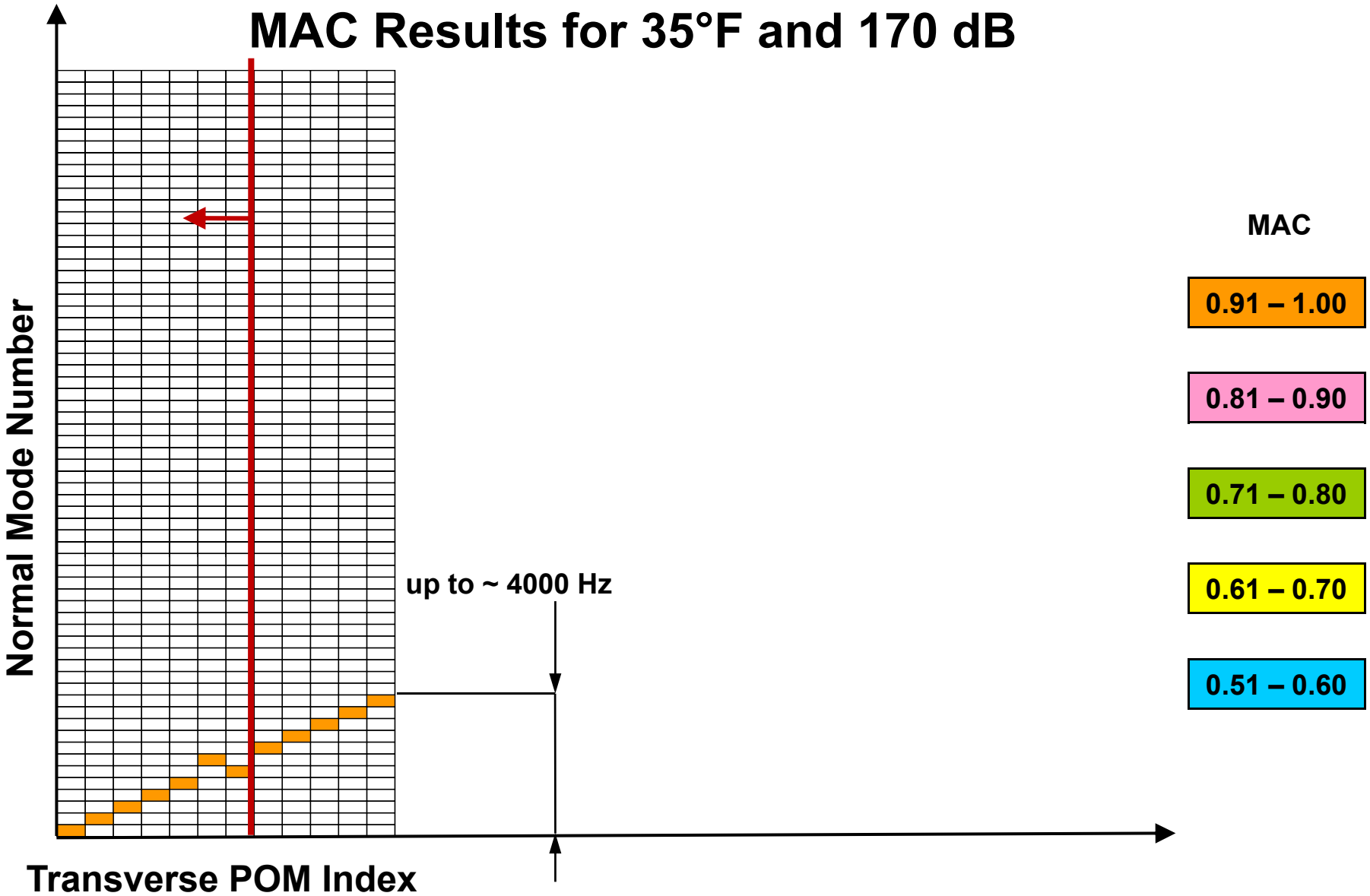
- **Application of POD/MAC procedure to loadings with time varying temperature and random pressure spatial distribution**

- **Verification and validation by comparison with**
  - Full order finite element results
  - Experimental data

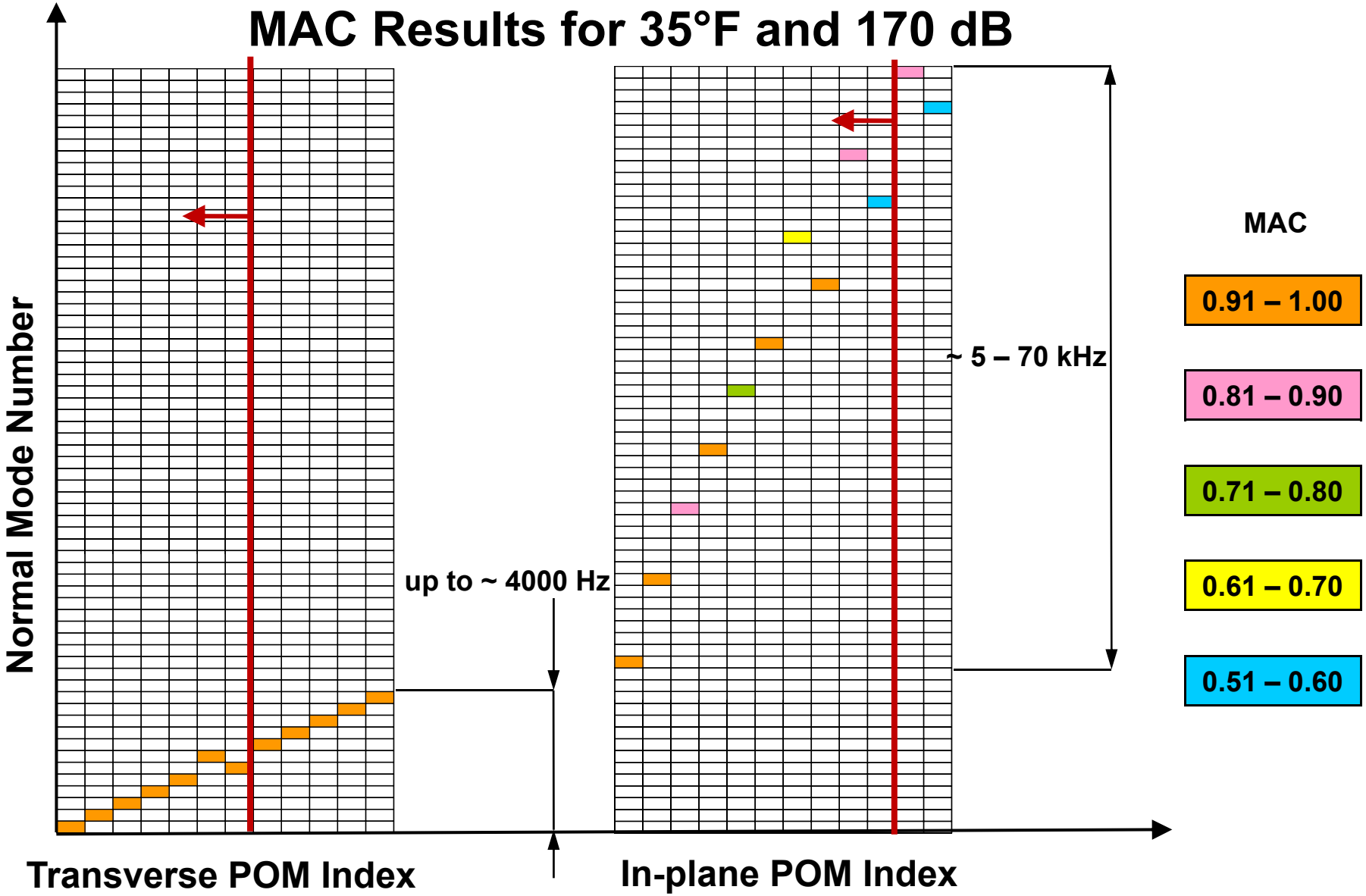
Backup Charts

Reduced Order Model Basis Selection

MAC Results for 35°F and 170 dB



Reduced Order Model Basis Selection

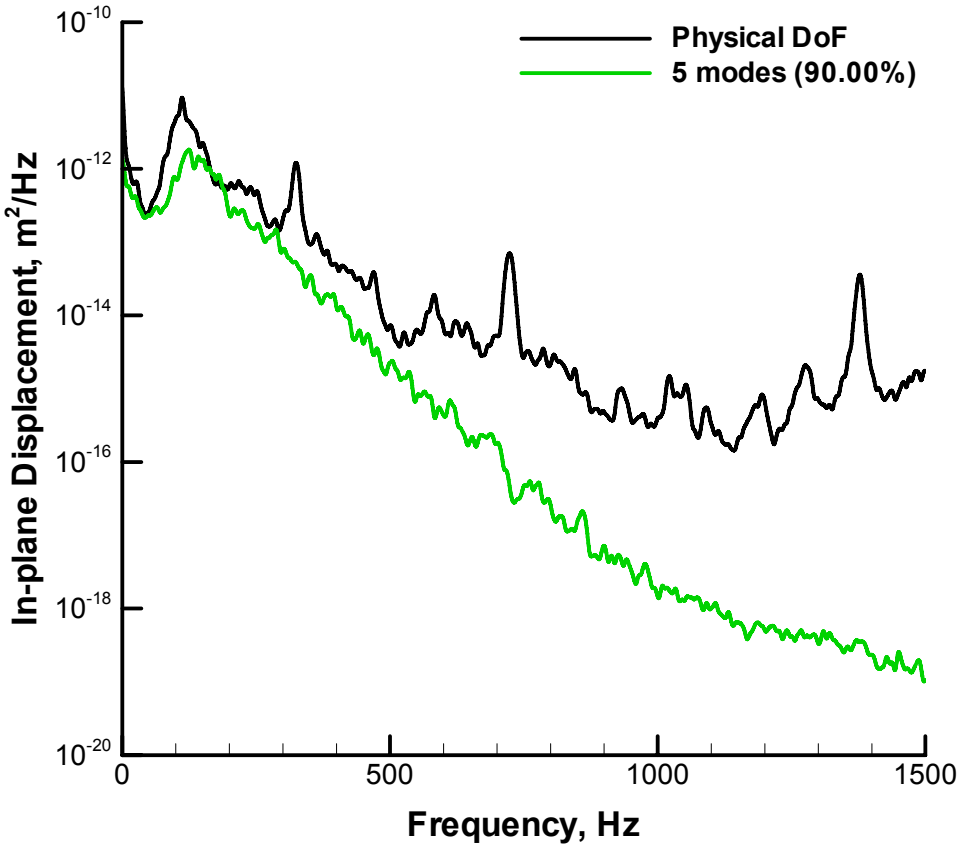


Robustness Study:

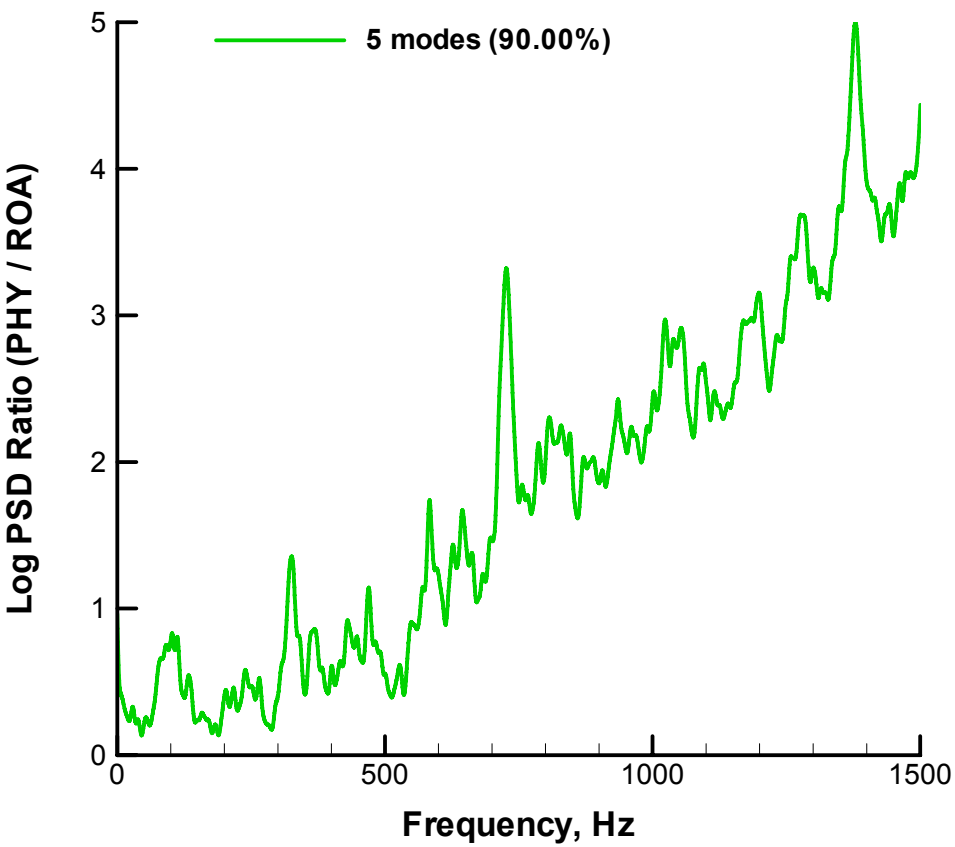
Nonlinear Response at a Reduced Acoustic Loading Level (35°F and 158 dB)

Reduced order analysis derived from 35°F and 170 dB loading level

In-plane response at quarter-span location



Error measure



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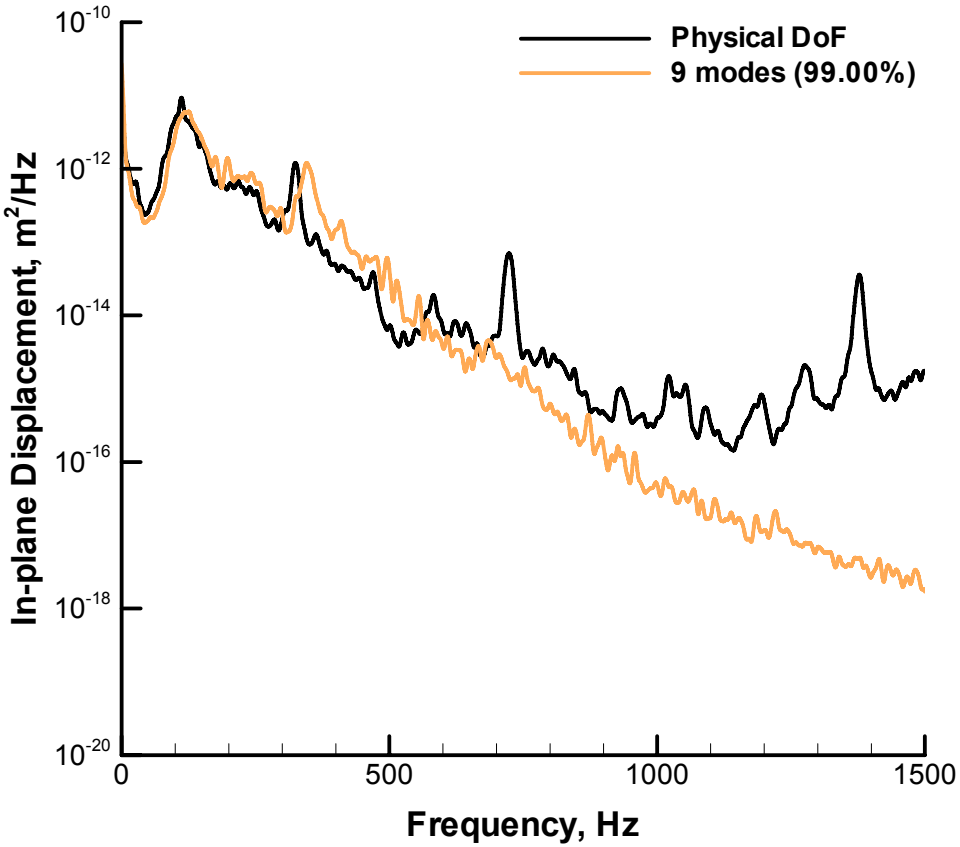
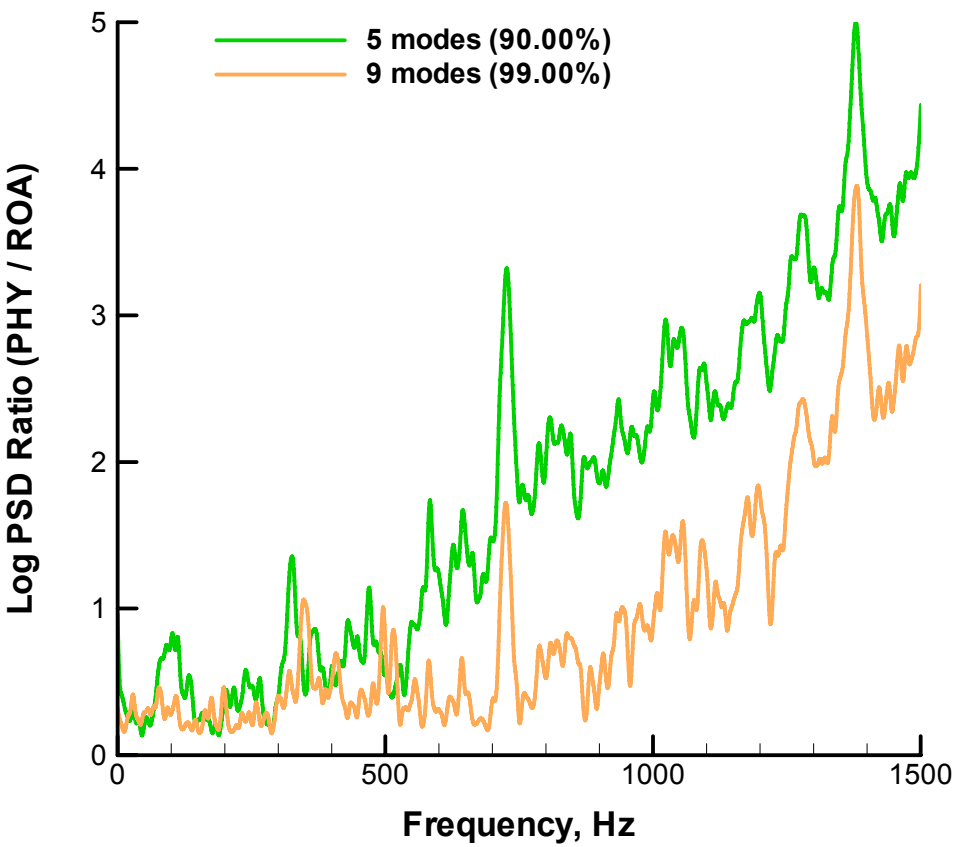
Nonlinear Response at a Reduced Acoustic Loading Level (35°F and 158 dB)

Reduced order analysis derived from 35°F and 170 dB loading level

In-plane response at quarter-span location



Error measure



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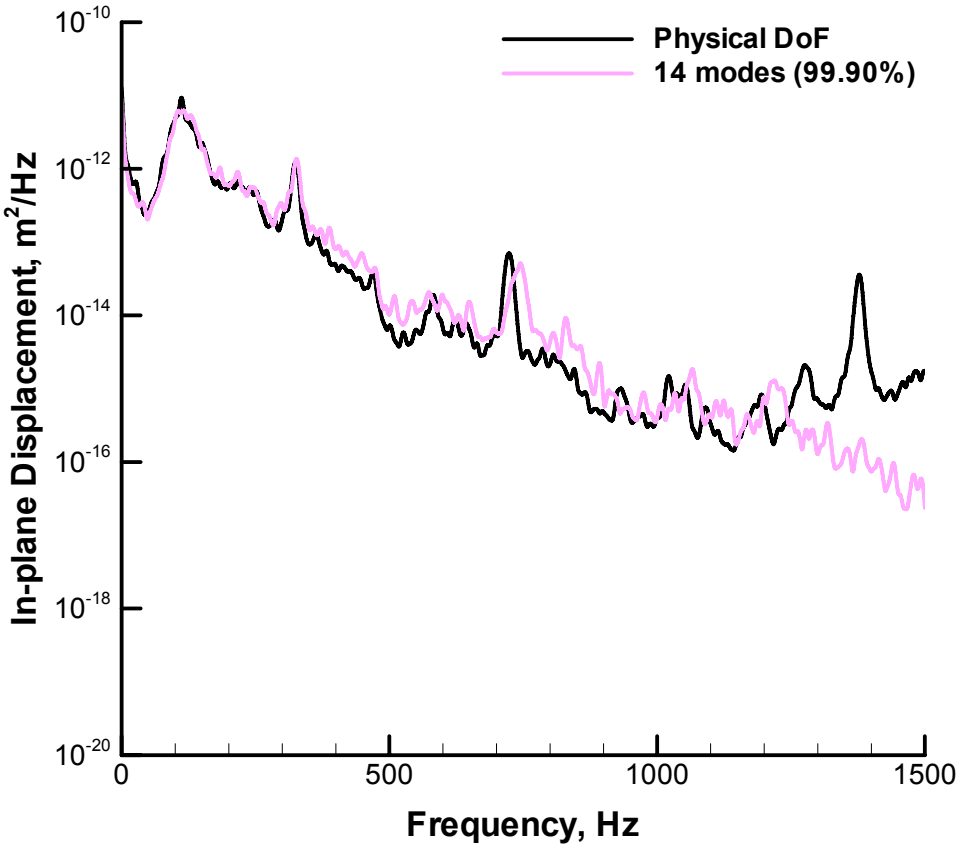
Nonlinear Response at a Reduced Acoustic Loading Level (35°F and 158 dB)

Reduced order analysis derived from 35°F and 170 dB loading level

In-plane response at quarter-span location



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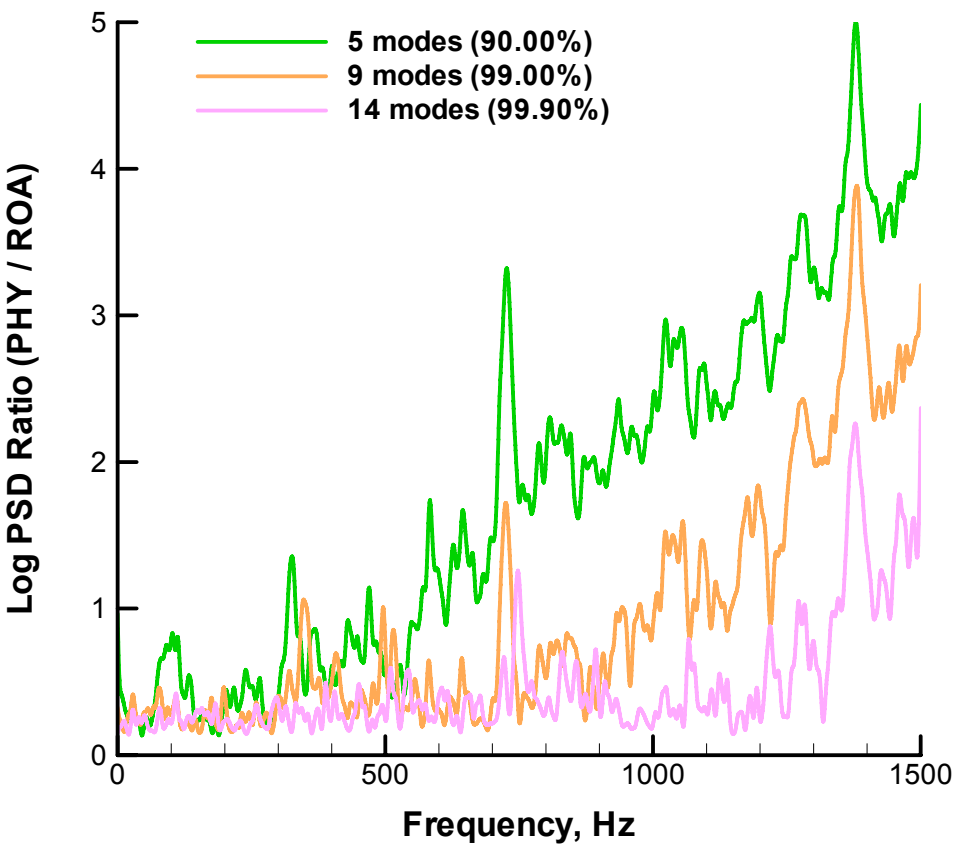
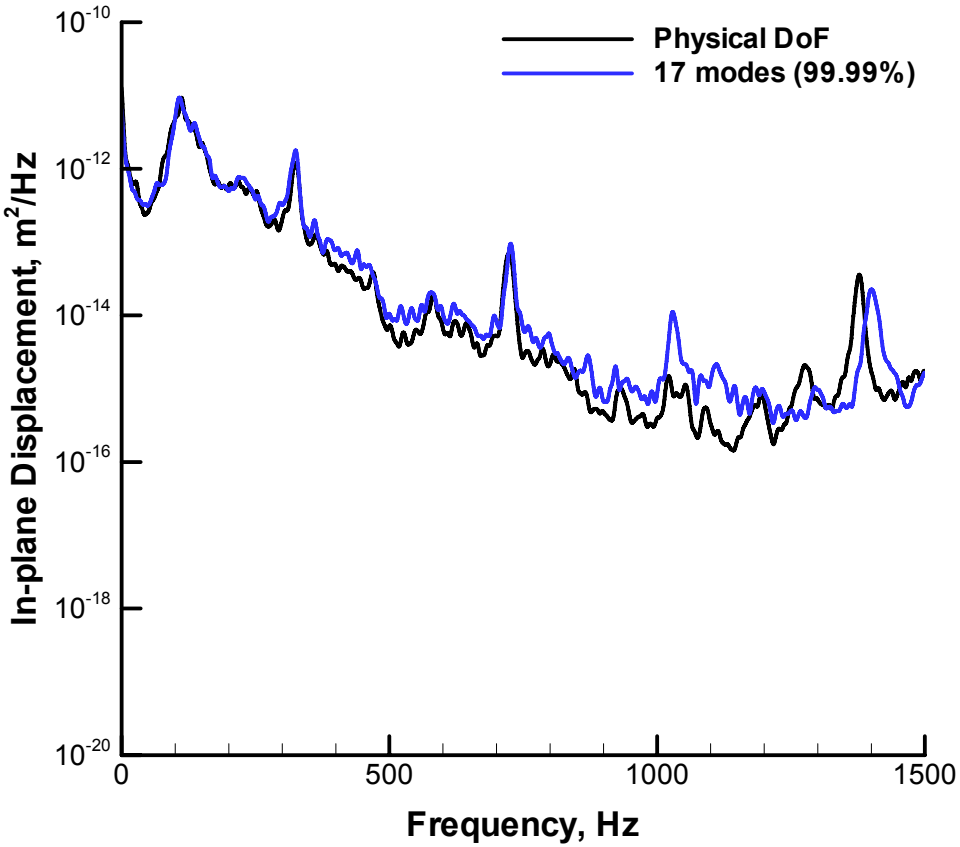


Robustness Study:

Nonlinear Response at a Reduced Acoustic Loading Level (35°F and 158 dB)

Reduced order analysis derived from 35°F and 170 dB loading level

In-plane response at quarter-span location



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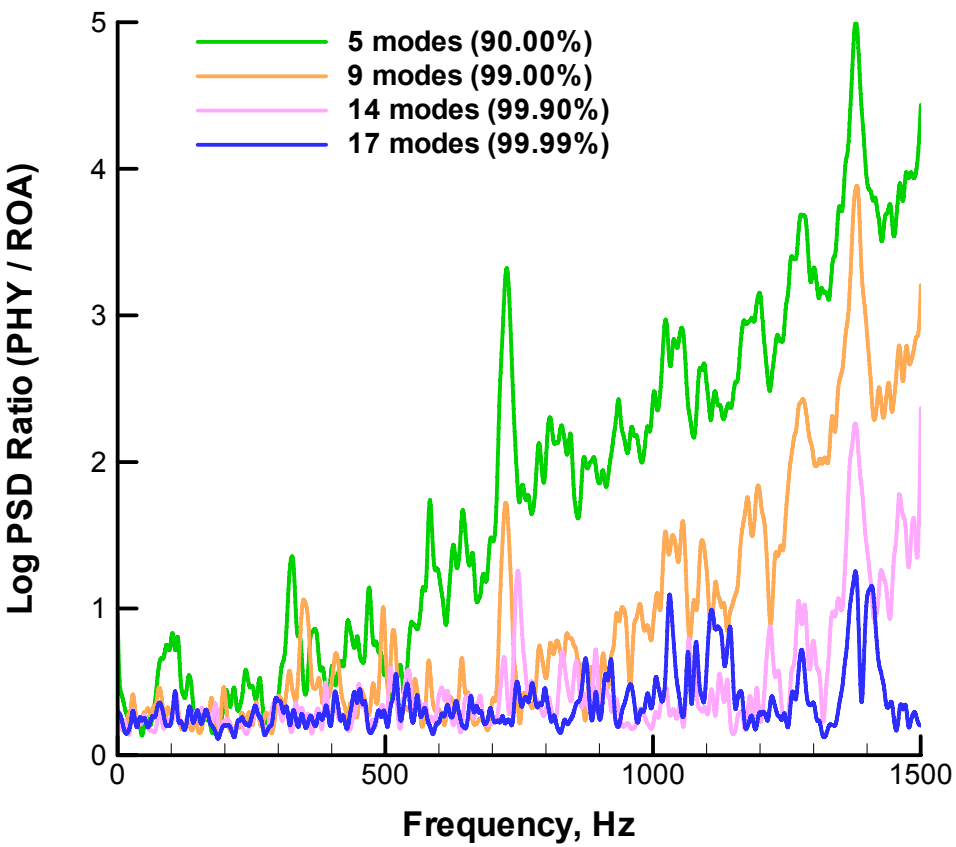
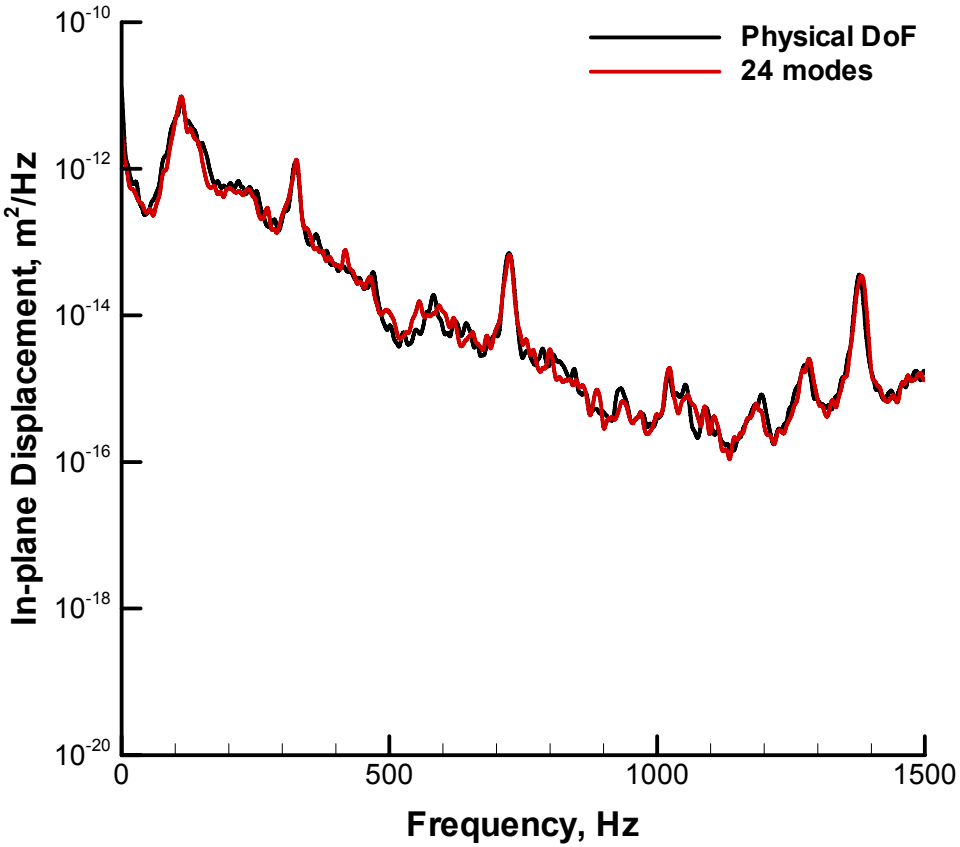


Robustness Study:

Nonlinear Response at a Reduced Acoustic Loading Level (35°F and 158 dB)

Reduced order analysis derived from 35°F and 170 dB loading level

In-plane response at quarter-span location



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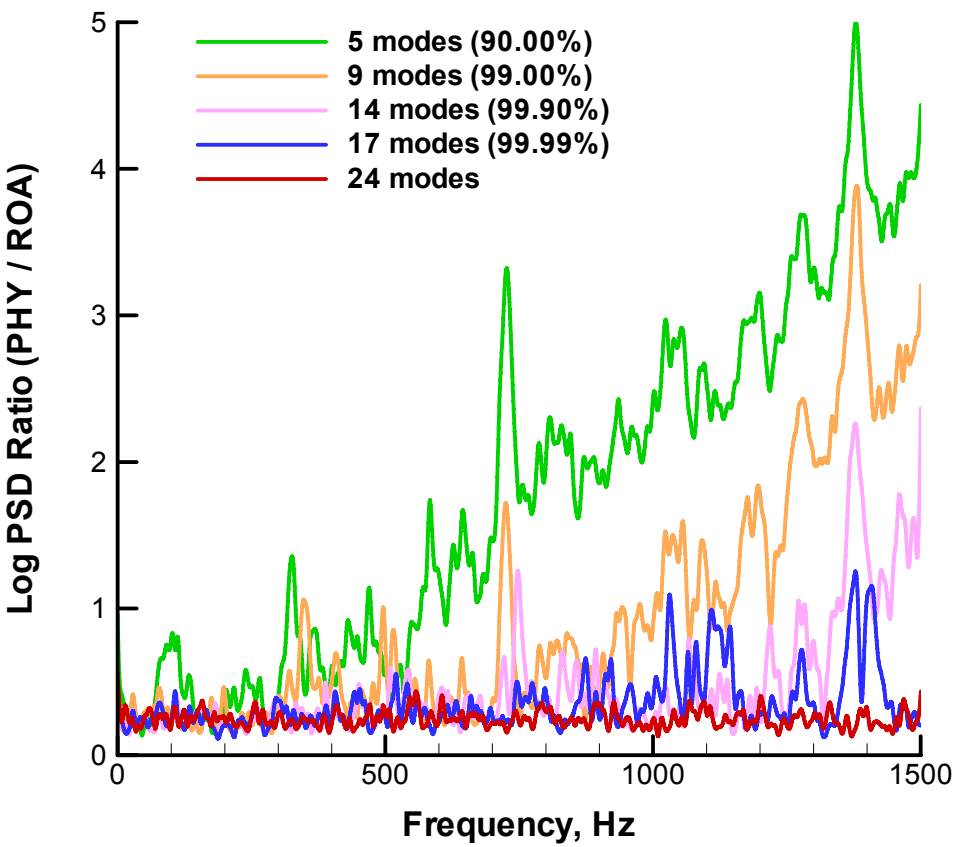
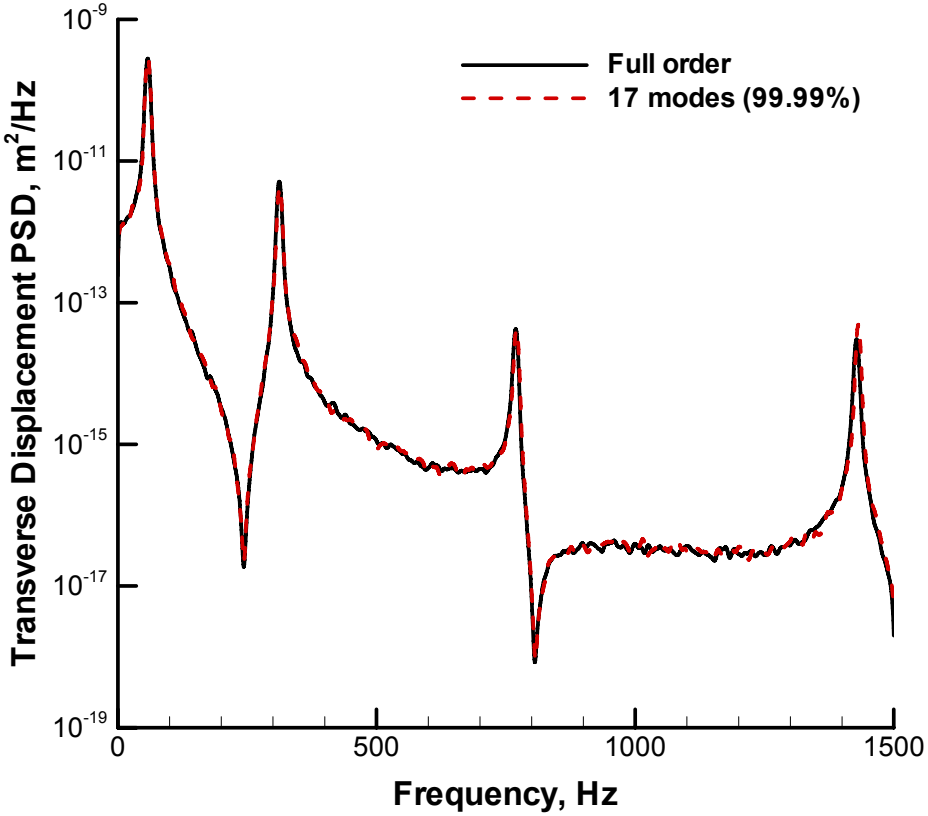


Robustness Study 2:

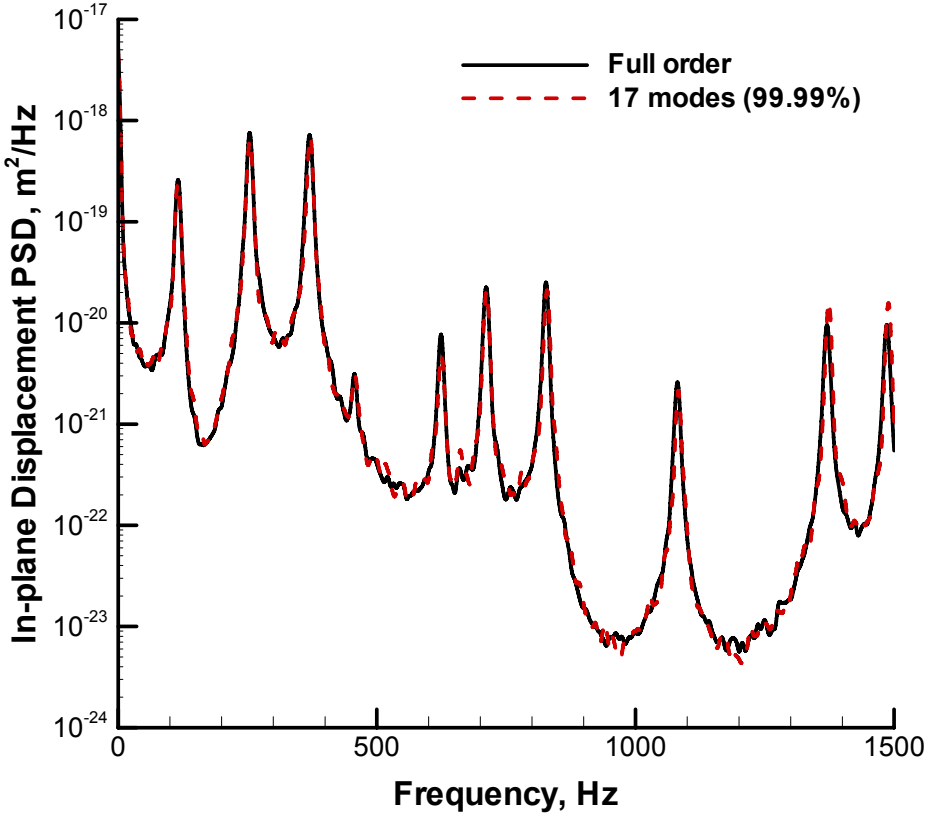
Response to a Low Level Acoustic Loading (32°F and 128 dB)

- **Reduced order analysis derived from 35°F and 170 dB loading level**

Quarter-span location



Transverse Response



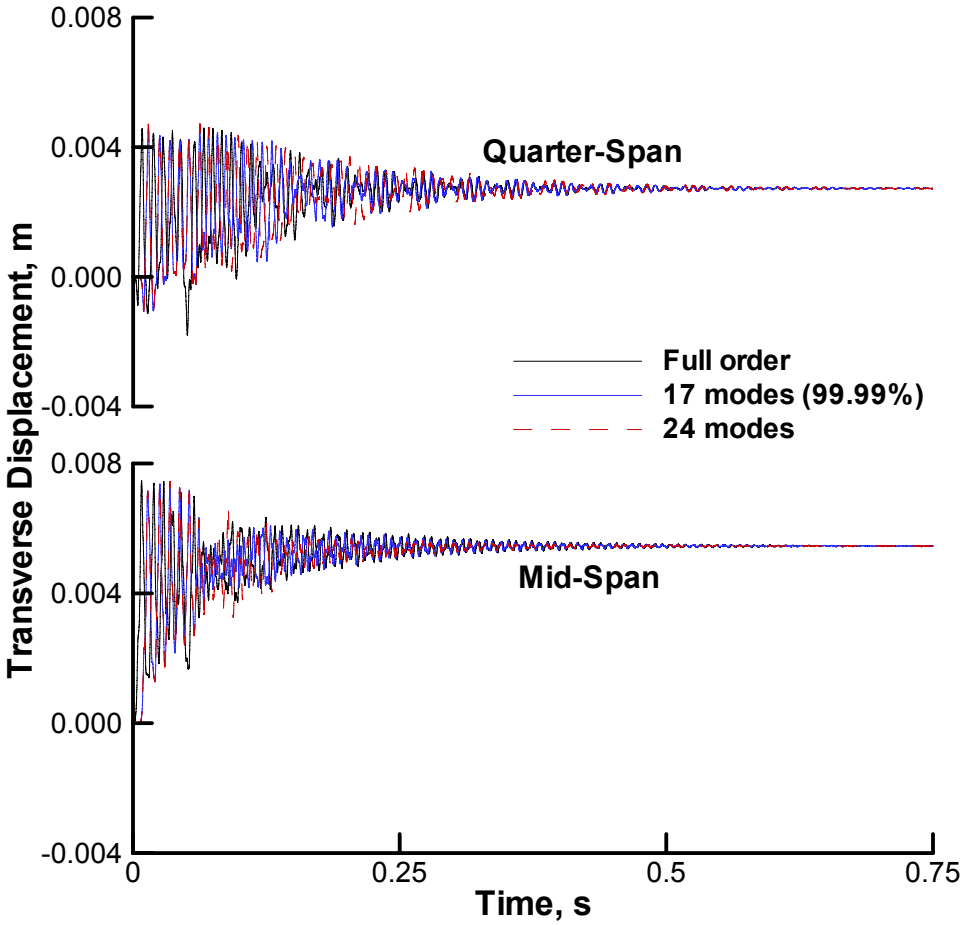
In-Plane Response

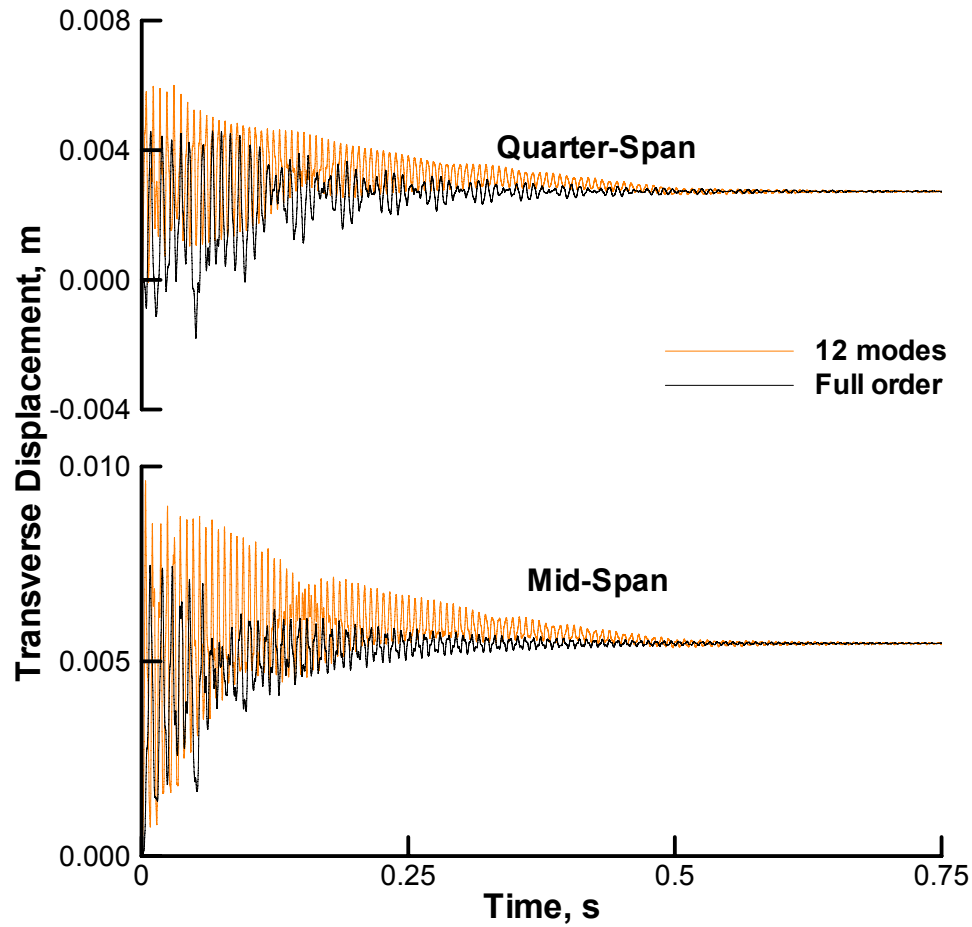
Robustness Study 3:

Dynamic Thermal Buckling (35°F)

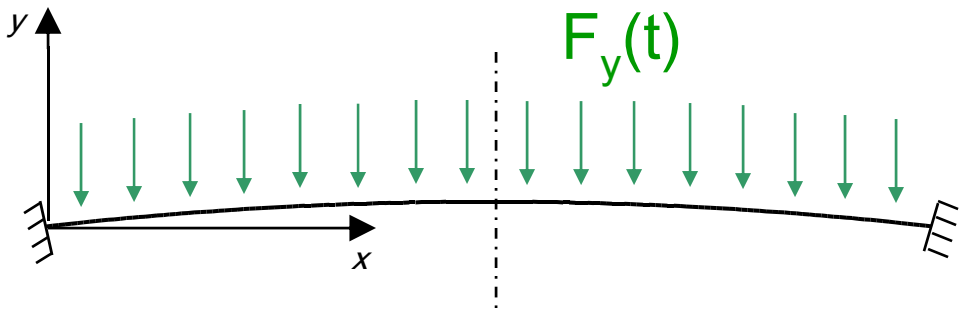
- **Reduced order analysis derived from 35°F and 170 dB loading level**

Quarter- and mid-span locations





Study Case – Aluminum Shallow Arch

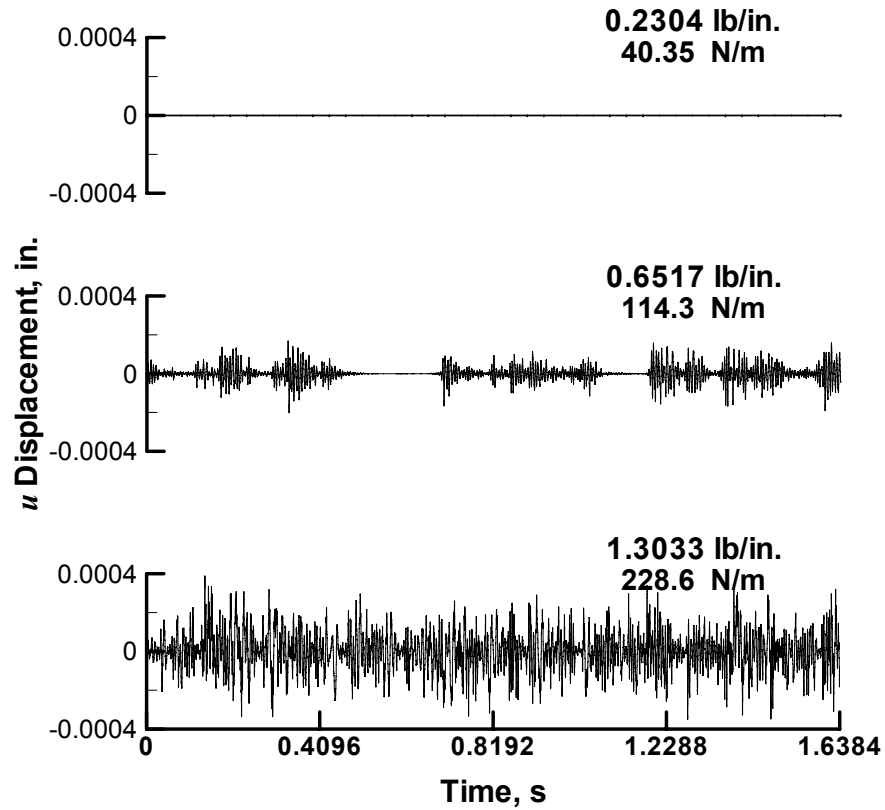


0.4572 m (projected) *x* 25.4 mm *x* 22.86 mm

2.165 m radius

144 B21 elements

Clamped Boundary Conditions



<u>Excitation</u>

- **Uniformly distributed, band-limited white noise normal load (0 – 1500 Hz)**

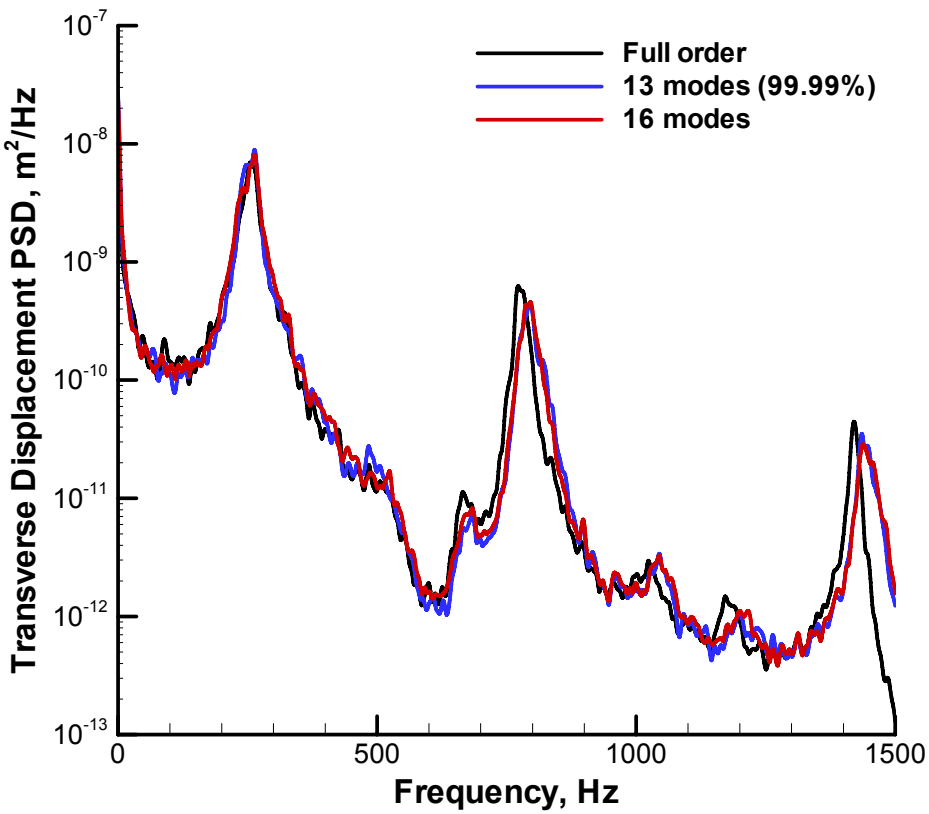
In-plane Displacement

Random Response

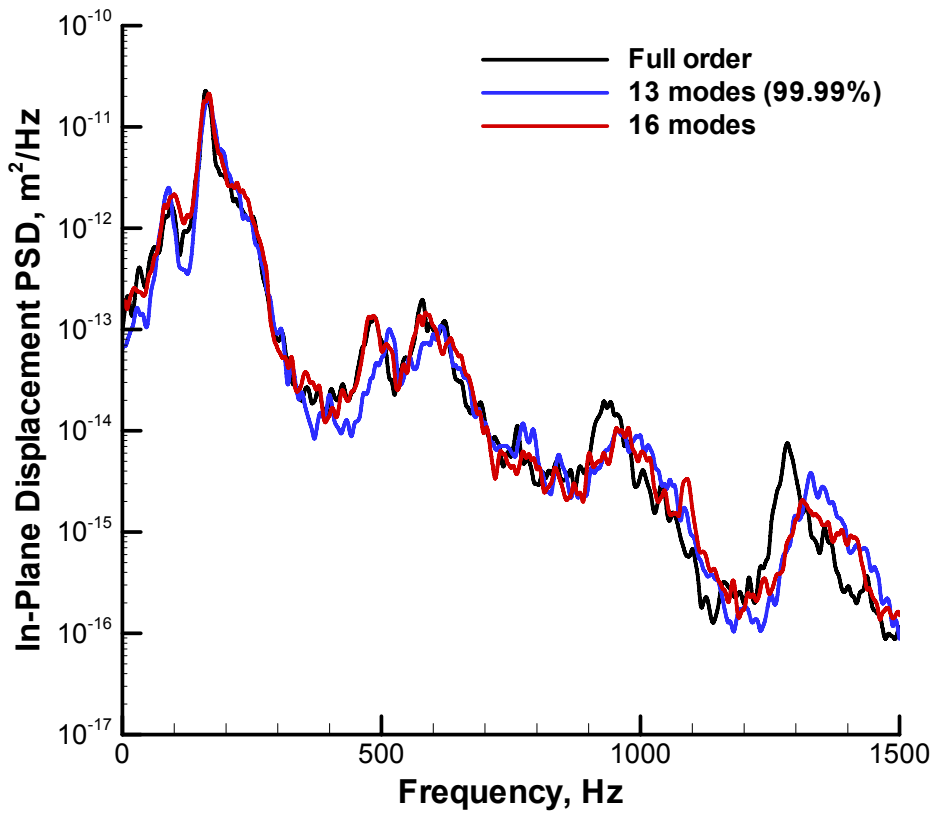
with Auto-parametric Resonance

Reduced Order Analysis (114.3 N/m)

Mid-span location



Transverse Response



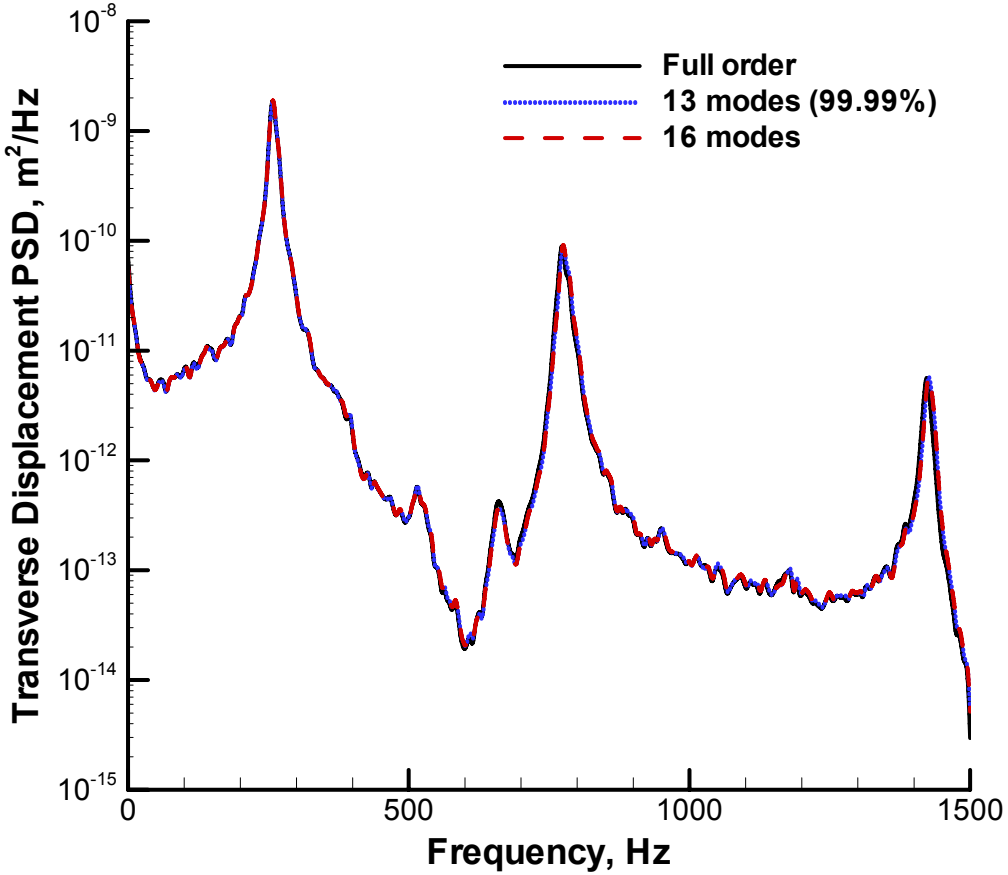
In-Plane Response

Robustness Study 4:

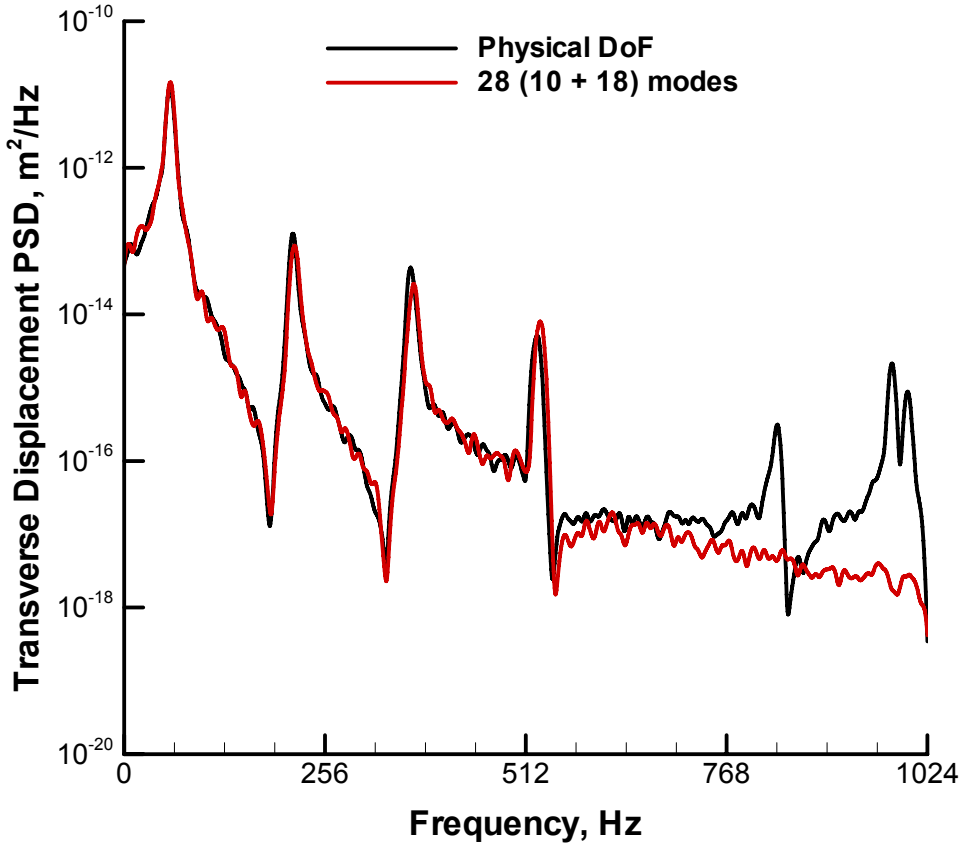
Nonlinear Response at a Reduced Distributed Loading Level (40.35 N/m)

- **Reduced order analysis derived from 114.3 N/m loading level**

Mid-span location



Generalization to Large 2D and 3D Structures



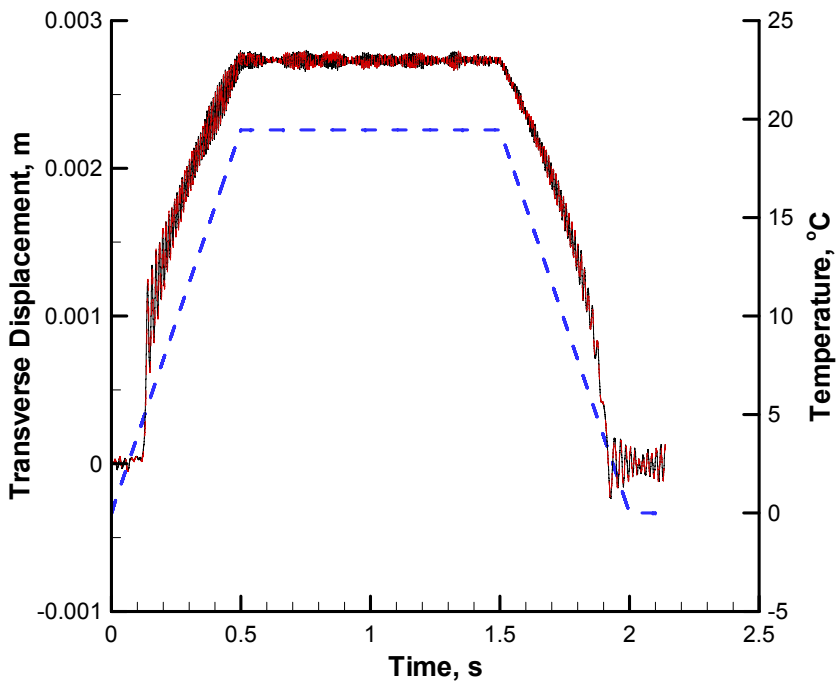
Flat Plate (approx. 55,000 DoF)

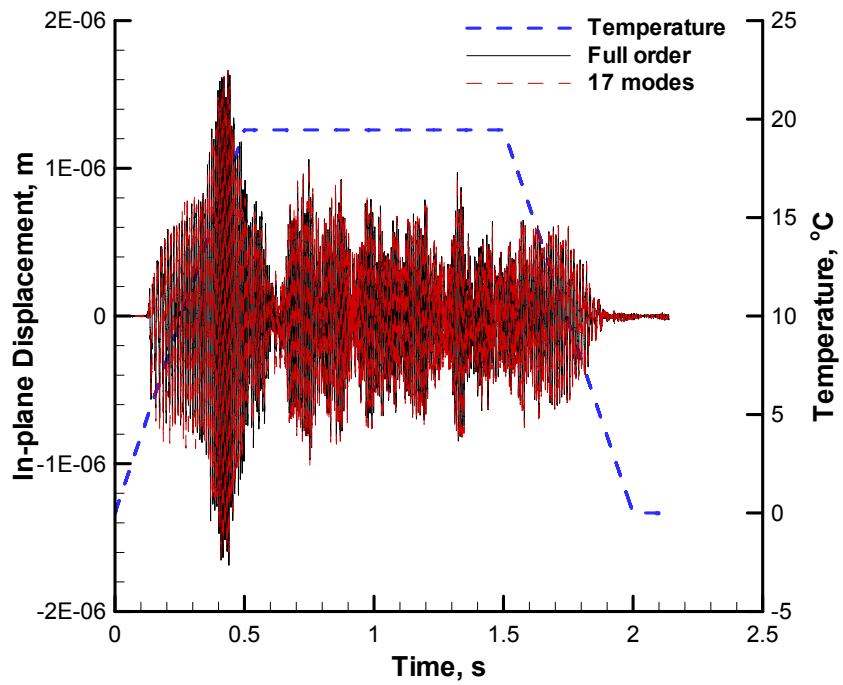
- POD performed at 160 dB (results shown at 106 dB)
- Transverse selection (POM participation ≥ 0.20%)
- In-plane selection (POM participation ≥ 1.00%)

RESULT: 10 TRANSVERSE +18 IN-PLANE MODES

NRA Year 2 Explorations

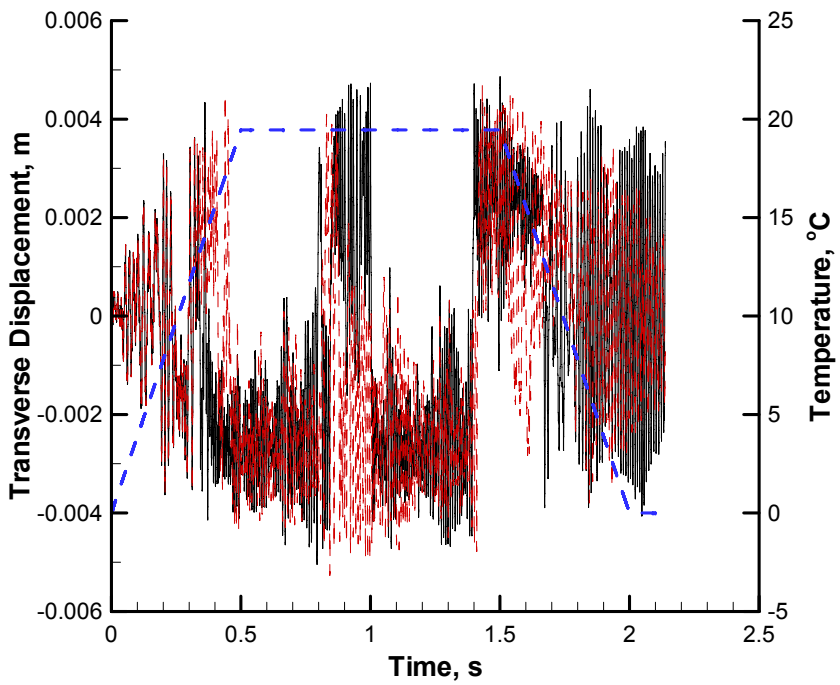
- **Apply POD/MAC procedure to loadings with time varying temperature and random pressure spatial distribution**
  - Reduced order analysis derived from 19.4 °C/ 170 dB loading level
  - Temperature profile 0→19.4→0 °C/ 128 dB (const.)
  - Results at the quarter-span location

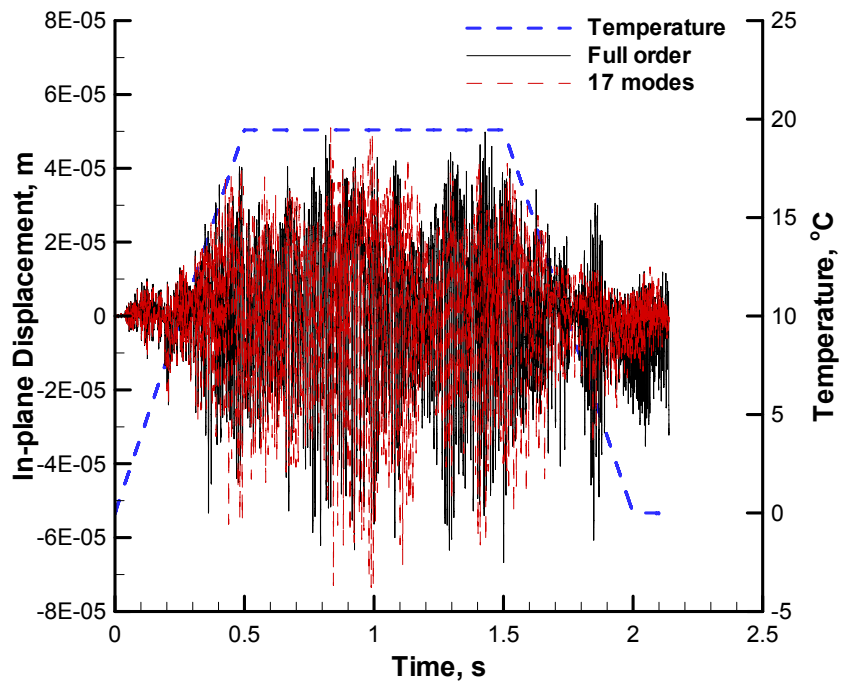




NRA Year 2 Explorations

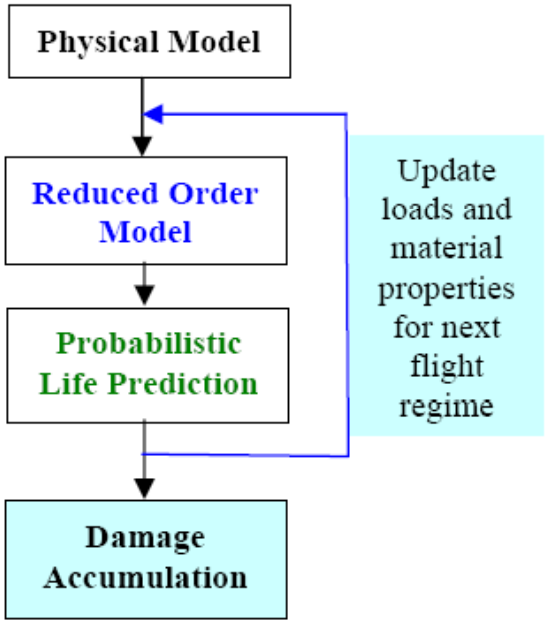
- **Apply POD/MAC procedure to loadings with time varying temperature and random pressure spatial distribution**
  - Reduced order analysis derived from 19.4 °C/ 170 dB loading level
  - Temperature profile 0→19.4→0 °C/ 158 dB (const.)
  - Results at the quarter-span location

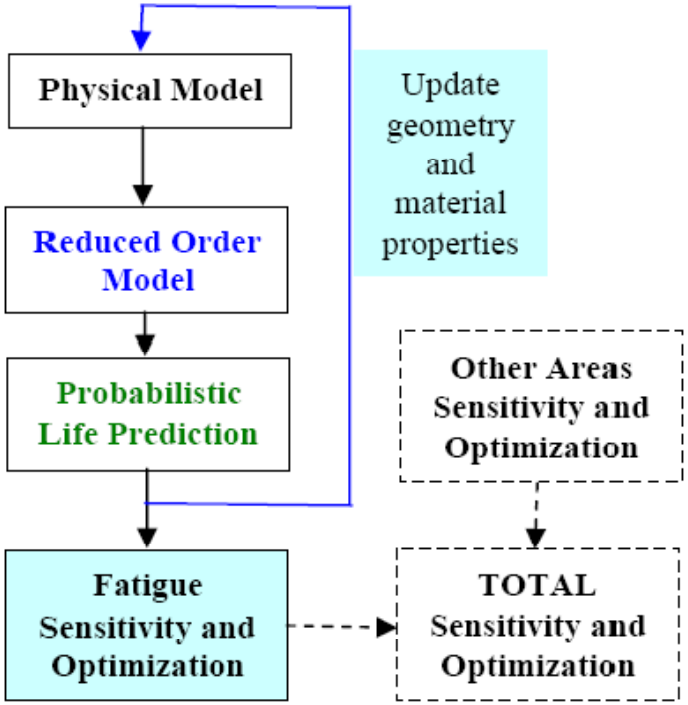




NRA Year 3 Activities

- Improved estimation of fatigue life utilizing non-zero mean stress damage accumulation models
- Full hypersonic mission cycle analysis - OR - Sensitivity and optimization

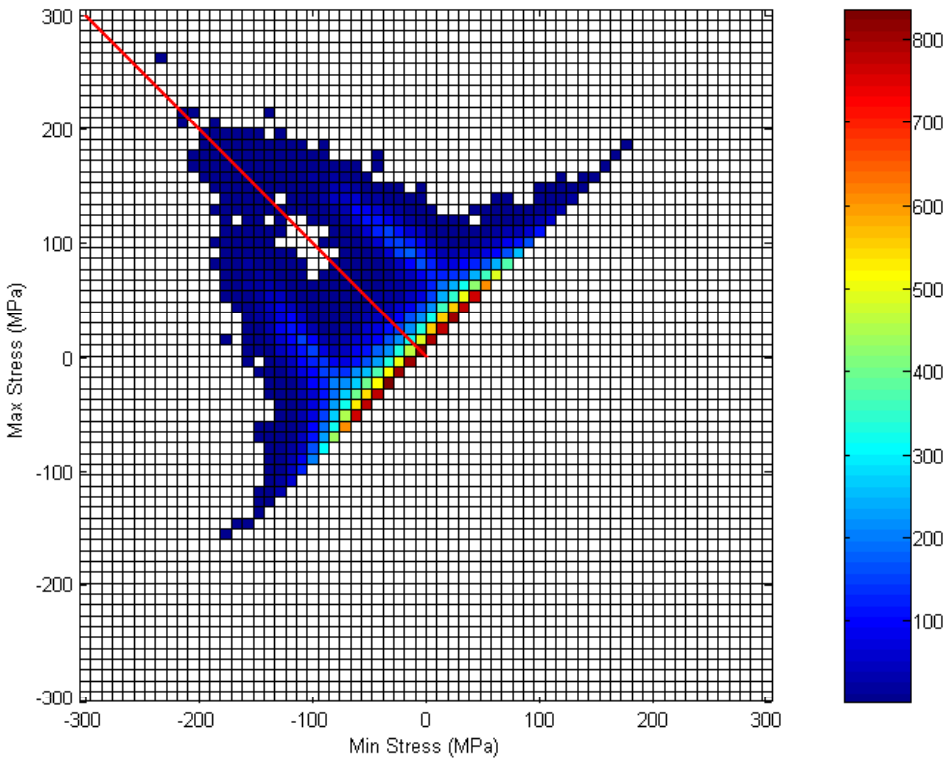




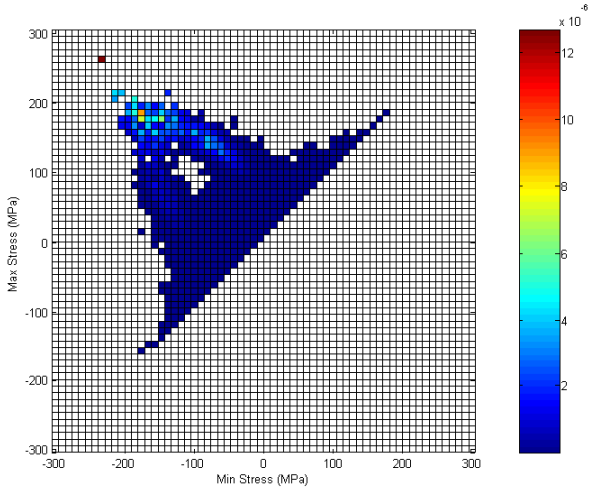
NRA Year 3 Explorations

- **Improved estimation of fatigue life utilizing non-zero mean stress damage accumulation models**

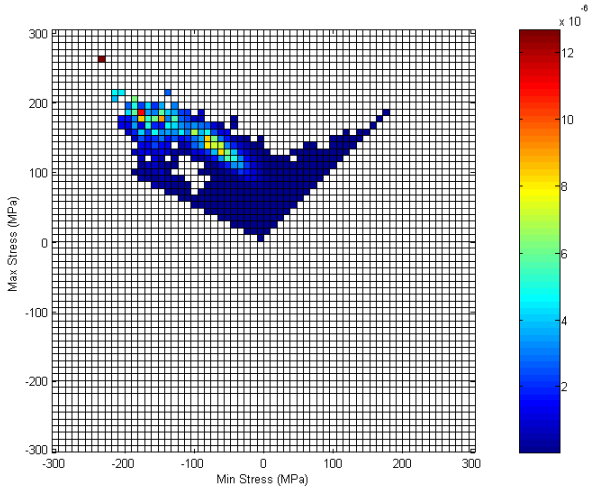
Intermittent Snap-through (19.4 °C/ 158 dB)



RFM



RFD – Morrow TFS



RFD – Standard Walker

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